12.3 Analyze Geometric Sequences and Series

Before	You studied arithmetic sequences and series.
Now	You will study geometric sequences and series.
Why?	So you can solve problems about sports tournaments, as in Ex. 58.

Key Vocabulary
- geometric sequence
- common ratio
- geometric series

In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by r.

EXAMPLE 1 Identify geometric sequences

Tell whether the sequence is geometric.

a. 4, 10, 18, 28, 40, . . .

b. 625, 125, 25, 5, 1, . . .

Solution

To decide whether a sequence is geometric, find the ratios of consecutive terms.

a. $\dfrac{a_2}{a_1} = \dfrac{10}{4} = \dfrac{5}{2}$ $\qquad \dfrac{a_3}{a_2} = \dfrac{18}{10} = \dfrac{9}{5}$ $\qquad \dfrac{a_4}{a_3} = \dfrac{28}{18} = \dfrac{14}{9}$ $\qquad \dfrac{a_5}{a_4} = \dfrac{40}{28} = \dfrac{10}{7}$

▶ The ratios are different, so the sequence is not geometric.

b. $\dfrac{a_2}{a_1} = \dfrac{125}{625} = \dfrac{1}{5}$ $\qquad \dfrac{a_3}{a_2} = \dfrac{25}{125} = \dfrac{1}{5}$ $\qquad \dfrac{a_4}{a_3} = \dfrac{5}{25} = \dfrac{1}{5}$ $\qquad \dfrac{a_5}{a_4} = \dfrac{1}{5}$

▶ Each ratio is $\dfrac{1}{5}$, so the sequence is geometric.

✓ **GUIDED PRACTICE** for Example 1

Tell whether the sequence is geometric. *Explain* why or why not.

1. 81, 27, 9, 3, 1, . . .

2. 1, 2, 6, 24, 120, . . .

3. −4, 8, −16, 32, −64, . . .

KEY CONCEPT *For Your Notebook*

Rule for a Geometric Sequence

Algebra The nth term of a geometric sequence with first term a_1 and common ratio r is given by:

$$a_n = a_1 r^{n-1}$$

Example The nth term of a geometric sequence with a first term of 3 and common ratio 2 is given by:

$$a_n = 3(2)^{n-1}$$