EXAMPLE 4 Write a rule given two terms

Two terms of an arithmetic sequence are $a_8 = 21$ and $a_{27} = 97$. Find a rule for the nth term.

Solution

STEP 1 **Write** a system of equations using $a_n = a_1 + (n - 1)d$ and substituting 27 for n (Equation 1) and then 8 for n (Equation 2).

$a_{27} = a_1 + (27 - 1)d \rightarrow 97 = a_1 + 26d$ **Equation 1**

$a_8 = a_1 + (8 - 1)d \rightarrow 21 = a_1 + 7d$ **Equation 2**

STEP 2 **Solve** the system.

$76 = 19d$ **Subtract.**

$4 = d$ **Solve for *d*.**

$97 = a_1 + 26(4)$ **Substitute for *d* in Equation 1.**

$-7 = a_1$ **Solve for a_1.**

STEP 3 **Find** a rule for a_n.

$a_n = a_1 + (n - 1)d$ **Write general rule.**

$= -7 + (n - 1)4$ **Substitute for a_1 and *d*.**

$= -11 + 4n$ **Simplify.**

GUIDED PRACTICE for Examples 2, 3, and 4

Write a rule for the nth term of the arithmetic sequence. Then find a_{20}.

2. 17, 14, 11, 8, . . .

3. $a_{11} = -57, d = -7$

4. $a_7 = 26, a_{16} = 71$

ARITHMETIC SERIES The expression formed by adding the terms of an arithmetic sequence is called an **arithmetic series**. The sum of the first n terms of an arithmetic series is denoted by S_n. To find a rule for S_n, you can write S_n in two different ways and add the results.

$$\begin{aligned} S_n &= a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + a_n \\ S_n &= a_n + (a_n - d) + (a_n - 2d) + \cdots + a_1 \\ \hline 2S_n &= (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n) \end{aligned}$$

You can conclude that $2S_n = n(a_1 + a_n)$, which leads to the following result.

KEY CONCEPT *For Your Notebook*

The Sum of a Finite Arithmetic Series

The sum of the first n terms of an arithmetic series is:

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

In words, S_n is the mean of the first and nth terms, multiplied by the number of terms.