INDEX OF SUMMATION The index of summation for a series does not have to be i—any letter can be used. Also, the index does not have to begin at 1. For instance, the index begins at 4 in the next example.

EXAMPLE 5 Find the sum of a series

AVOID ERRORS
Be sure to use the correct lower and upper limits of summation when finding the sum of a series.

Find the sum of the series.

$$\sum_{k=4}^{8} (3 + k^2) = (3 + 4^2) + (3 + 5^2) + (3 + 6^2) + (3 + 7^2) + (3 + 8^2)$$
$$= 19 + 28 + 39 + 52 + 67$$
$$= 205$$

SPECIAL FORMULAS For series with many terms, finding the sum by adding the terms can be tedious. Below are formulas you can use to find the sums of three special types of series.

KEY CONCEPT *For Your Notebook*

Formulas for Special Series

Sum of n terms of 1	Sum of first n positive integers	Sum of squares of first n positive integers
$\sum_{i=1}^{n} 1 = n$	$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

EXAMPLE 6 Use a formula for a sum

RETAIL DISPLAYS How many apples are in the stack in Example 3 on page 795?

Solution

From Example 3 you know that the ith term of the series is given by $a_i = i^2$ where $i = 1, 2, 3, \ldots, 7$. Using summation notation and the third formula listed above, you can find the total number of apples as follows:

$$1^2 + 2^2 + \cdots + 7^2 = \sum_{i=1}^{7} i^2 = \frac{7(7+1)(2 \cdot 7 + 1)}{6} = \frac{7(8)(15)}{6} = 140$$

▶ There are 140 apples in the stack. Check this by actually adding the number of apples in each of the seven layers.

✓ GUIDED PRACTICE for Examples 5 and 6

Find the sum of the series.

10. $\sum_{i=1}^{5} 8i$ **11.** $\sum_{k=3}^{7} (k^2 - 1)$ **12.** $\sum_{i=1}^{34} 1$ **13.** $\sum_{n=1}^{6} n$

14. **WHAT IF?** Suppose there are 9 layers in the apple stack in Example 3. How many apples are in the stack?