KEY CONCEPT *For Your Notebook*

Series and Summation Notation

When the terms of a sequence are added together, the resulting expression is a **series**. A series can be finite or infinite.

Finite series: $2 + 4 + 6 + 8$ **Infinite series:** $2 + 4 + 6 + 8 + \cdots$

You can use **summation notation** to write a series. For example, the two series above can be written in summation notation as follows:

$$2 + 4 + 6 + 8 = \sum_{i=1}^{4} 2i \qquad 2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i$$

For both series, the *index of summation* is i and the *lower limit of summation* is 1. The *upper limit of summation* is 4 for the finite series and ∞ (infinity) for the infinite series. Summation notation is also called **sigma notation** because it uses the uppercase Greek letter *sigma*, written Σ.

READING
When written in summation notation, this series is read as "the sum of $2i$ for values of i from 1 to 4."

EXAMPLE 4 Write series using summation notation

Write the series using summation notation.

a. $25 + 50 + 75 + \cdots + 250$

b. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots$

Solution

a. Notice that the first term is 25(1), the second is 25(2), the third is 25(3), and the last is 25(10). So, the terms of the series can be written as:

$$a_i = 25i \text{ where } i = 1, 2, 3, \ldots, 10$$

The lower limit of summation is 1 and the upper limit of summation is 10.

▶ The summation notation for the series is $\sum_{i=1}^{10} 25i$.

b. Notice that for each term the denominator of the fraction is 1 more than the numerator. So, the terms of the series can be written as:

$$a_i = \frac{i}{i+1} \text{ where } i = 1, 2, 3, 4, \ldots$$

The lower limit of summation is 1 and the upper limit of summation is infinity.

▶ The summation notation for the series is $\sum_{i=1}^{\infty} \frac{i}{i+1}$.

✓ GUIDED PRACTICE for Example 4

Write the series using summation notation.

6. $5 + 10 + 15 + \cdots + 100$

7. $\frac{1}{2} + \frac{4}{5} + \frac{9}{10} + \frac{16}{17} + \cdots$

8. $6 + 36 + 216 + 1296 + \cdots$

9. $5 + 6 + 7 + \cdots + 12$