

## Extension

Use after Lesson 11.3

# Approximate Binomial Distributions and Test Hypotheses



**GOAL** Use normal distributions to approximate binomial distributions.

In Lesson 10.6, you found probabilities related to a binomial distribution using the formula  $P(k) = {}_n C_k p^k (1-p)^{n-k}$ . However, it can be tedious to use this formula when the number of probabilities to compute is large. In such cases, you may be able to use a normal distribution to approximate the binomial distribution.

### KEY CONCEPT

*For Your Notebook*

#### Normal Approximation of a Binomial Distribution

Consider the binomial distribution consisting of  $n$  trials with probability  $p$  of success on each trial. If  $np \geq 5$  and  $n(1-p) \geq 5$ , then the binomial distribution can be approximated by a normal distribution with the following mean and standard deviation.

$$\text{Mean: } \bar{x} = np$$

$$\text{Standard Deviation: } \sigma = \sqrt{np(1-p)}$$

### EXAMPLE 1 Find a binomial probability

**SURVEYS** According to a survey conducted by the Harris Poll, 24% of adults in the United States say that their favorite leisure-time activity is reading. You are conducting a random survey of 250 adults. What is the probability that you will find at most 53 adults who say that reading is their favorite leisure-time activity?

#### Solution

The number  $x$  of adults in your survey who say reading is their favorite leisure-time activity has a binomial distribution with  $n = 250$  and  $p = 0.24$ . To solve the problem using the binomial probability formula, you would have to calculate the following:

$$P(x \leq 53) = P(x = 0) + P(x = 1) + P(x = 2) + \cdots + P(x = 53)$$

This would be tedious. Instead, you can approximate the answer using a normal distribution with the mean and standard deviation given below.

$$\bar{x} = np = 250(0.24) = 60$$

Find mean.

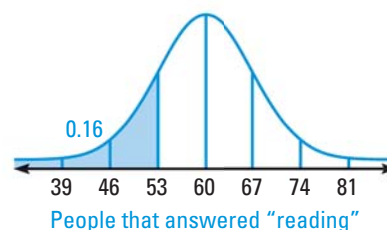
$$\sigma = \sqrt{np(1-p)} = \sqrt{250(0.24)(0.76)} \approx 7$$

Find standard deviation.

For this normal distribution, 53 is about one standard deviation to the left of the mean. Therefore:

$$P(x \leq 53) \approx 0.0015 + 0.0235 + 0.135 = 0.16$$

► The probability that at most 53 of the people surveyed say reading is their favorite leisure-time activity is about 0.16.



#### CHECK REASONABLENESS

In Example 1, note that

$$np = 60 \geq 5$$

and that

$$n(1-p) = 190 \geq 5.$$

So, it is reasonable to use a normal approximation.