EXAMPLE 3 Construct a binomial distribution

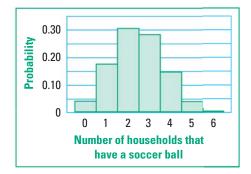
SPORTS SURVEYS According to a survey, about 41% of U.S. households have a soccer ball. Suppose you ask 6 randomly chosen U.S. households whether they have a soccer ball. Draw a histogram of the binomial distribution for your survey.

Solution

The probability that a randomly selected household has a soccer ball is p = 0.41. Because you survey 6 households, n = 6.

AVOID ERRORS

You can check your calculations for a binomial distribution by adding all the probabilities. The sum should always be 1. $P(k = 0) = {}_{6}C_{0}(0.41)^{0}(0.59)^{6} \approx 0.042$ $P(k = 1) = {}_{6}C_{1}(0.41)^{1}(0.59)^{5} \approx 0.176$ $P(k = 2) = {}_{6}C_{2}(0.41)^{2}(0.59)^{4} \approx 0.306$ $P(k = 3) = {}_{6}C_{3}(0.41)^{3}(0.59)^{3} \approx 0.283$ $P(k = 4) = {}_{6}C_{4}(0.41)^{4}(0.59)^{2} \approx 0.148$ $P(k = 5) = {}_{6}C_{5}(0.41)^{5}(0.59)^{1} \approx 0.041$ $P(k = 6) = {}_{6}C_{6}(0.41)^{6}(0.59)^{0} \approx 0.005$



A histogram of the distribution is shown.

Animated Algebra at classzone.com

EXAMPLE 4 Interpret a binomial distribution

Use the binomial distribution in Example 3 to answer each question.

- a. What is the most likely outcome of the survey?
- b. What is the probability that at most 2 households have a soccer ball?

Solution

- **a.** The most likely outcome of the survey is the value of k for which P(k) is greatest. This probability is greatest for k = 2. So, the most likely outcome is that 2 of the 6 households have a soccer ball.
- **b.** The probability that at most 2 households have a soccer ball is:

 $P(k \le 2) = P(k = 2) + P(k = 1) + P(k = 0)$ $\approx 0.306 + 0.176 + 0.042$ ≈ 0.524

▶ So, the probability is about 52%.

GUIDED PRACTICE for Examples 3 and 4

In Sweden, 61% of households have a soccer ball. Suppose you ask 6 randomly chosen Swedish households whether they have a soccer ball.

- 3. Draw a histogram showing the binomial distribution for your survey.
- **4.** What is the most likely outcome of your survey? What is the probability that at most 2 households you survey have a soccer ball?