EXAMPLE 7 Find a coefficient in an expansion

Find the coefficient of x^4 in the expansion of $(3x + 2)^{10}$.

Solution

From the binomial theorem, you know the following:

$$(3x + 2)^{10} = {}_{10}C_0(3x)^{10}(2)^0 + {}_{10}C_1(3x)^9(2)^1 + \cdots + {}_{10}C_{10}(3x)^0(2)^{10}$$

Each term in the expansion has the form ${}_{10}C_r(3x)^{10-r}(2)^r$. The term containing x^4 occurs when $r = 6$:

$${}_{10}C_6(3x)^4(2)^6 = (210)(81x^4)(64) = 1{,}088{,}640x^4$$

▶ The coefficient of x^4 is 1,088,640.

✓ **GUIDED PRACTICE** for Example 7

11. Find the coefficient of x^5 in the expansion of $(x - 3)^7$.

12. Find the coefficient of x^3 in the expansion of $(2x + 5)^8$.

10.2 EXERCISES

SKILL PRACTICE

1. VOCABULARY Copy and complete: The binomial expansion of $(a + b)^n$ is given by the __?__.

2. WRITING *Explain* the difference between permutations and combinations.

EXAMPLES
1, 2, and 3
on pp. 690–691
for Exs. 3–18

COMBINATIONS Find the number of combinations.

3. ${}_5C_2$ **4.** ${}_{10}C_3$ **5.** ${}_9C_6$ **6.** ${}_8C_2$

7. ${}_{11}C_{11}$ **8.** ${}_{12}C_4$ **9.** ${}_7C_5$ **10.** ${}_{14}C_6$

ERROR ANALYSIS *Describe* and correct the error in finding the number of combinations.

11.
$${}_6C_2 = \frac{6!}{(6-2)!} = \frac{720}{24} = 30 \quad ✗$$

12.
$${}_8C_3 = \frac{8!}{3!} = \frac{40{,}320}{6} = 6720 \quad ✗$$

CARD HANDS Find the number of possible 5-card hands that contain the cards specified. The cards are taken from a standard 52-card deck.

13. 5 face cards (kings, queens, or jacks)   **14.** 4 kings and 1 other card

15. 1 ace and 4 cards that are not aces   **16.** 5 hearts or 5 diamonds

17. At most 1 queen   **18.** At least 1 spade