BINOMIAL EXPANSIONS There is an important relationship between powers of binomials and combinations. The numbers in Pascal's triangle can be used to find coefficients in binomial expansions. For example, the coefficients in the expansion of $(a + b)^4$ are the numbers of combinations in the row of Pascal's triangle for $n = 4$:

$$(a + b)^4 = \underset{_4C_0}{1}a^4 + \underset{_4C_1}{4}a^3b + \underset{_4C_2}{6}a^2b^2 + \underset{_4C_3}{4}ab^3 + \underset{_4C_4}{1}b^4$$

This result is generalized in the **binomial theorem**.

KEY CONCEPT — *For Your Notebook*

Binomial Theorem

For any positive integer n, the binomial expansion of $(a + b)^n$ is:

$$(a + b)^n = {_nC_0}a^nb^0 + {_nC_1}a^{n-1}b^1 + {_nC_2}a^{n-2}b^2 + \cdots + {_nC_n}a^0b^n$$

Notice that each term in the expansion of $(a + b)^n$ has the form ${_nC_r}\,a^{n-r}b^r$ where r is an integer from 0 to n.

EXAMPLE 5 Expand a power of a binomial sum

Use the binomial theorem to write the binomial expansion.

$$\begin{aligned}(x^2 + y)^3 &= {_3C_0}(x^2)^3y^0 + {_3C_1}(x^2)^2y^1 + {_3C_2}(x^2)^1y^2 + {_3C_3}(x^2)^0y^3\\ &= (1)(x^6)(1) + (3)(x^4)(y) + (3)(x^2)(y^2) + (1)(1)(y^3)\\ &= x^6 + 3x^4y + 3x^2y^2 + y^3\end{aligned}$$

POWERS OF BINOMIAL DIFFERENCES To expand a power of a binomial difference, you can rewrite the binomial as a sum. The resulting expansion will have terms whose signs alternate between + and −.

AVOID ERRORS

When a binomial has a term or terms with a coefficient other than 1, the coefficients of the binomial expansion are not the same as the corresponding row of Pascal's triangle.

EXAMPLE 6 Expand a power of a binomial difference

Use the binomial theorem to write the binomial expansion.

$$\begin{aligned}(a - 2b)^4 &= [a + (-2b)]^4\\ &= {_4C_0}a^4(-2b)^0 + {_4C_1}a^3(-2b)^1 + {_4C_2}a^2(-2b)^2 + {_4C_3}a^1(-2b)^3 + {_4C_4}a^0(-2b)^4\\ &= (1)(a^4)(1) + (4)(a^3)(-2b) + (6)(a^2)(4b^2) + (4)(a)(-8b^3) + (1)(1)(16b^4)\\ &= a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4\end{aligned}$$

✓ GUIDED PRACTICE for Examples 5 and 6

Use the binomial theorem to write the binomial expansion.

7. $(x + 3)^5$ **8.** $(a + 2b)^4$ **9.** $(2p - q)^4$ **10.** $(5 - 2y)^3$