10.2 Use Combinations and the Binomial Theorem

TEKS *a.1, a.2*

Before You used the counting principle and permutations.

Now You will use combinations and the binomial theorem.

Why? So you can find ways to form a set, as in Example 2.

Key Vocabulary
- **combination**
- **Pascal's triangle**
- **binomial theorem**

In Lesson 10.1, you learned that order is important for some counting problems. For other counting problems, order is not important. For instance, if you purchase a package of trading cards, the order of the cards inside the package is not important. A **combination** is a selection of r objects from a group of n objects where the order is not important.

KEY CONCEPT *For Your Notebook*

Combinations of n Objects Taken r at a Time

The number of combinations of r objects taken from a group of n distinct objects is denoted by ${}_nC_r$ and is given by this formula:

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

EXAMPLE 1 Find combinations

CARDS A standard deck of 52 playing cards has 4 suits with 13 different cards in each suit.

a. If the order in which the cards are dealt is not important, how many different 5-card hands are possible?

b. In how many 5-card hands are all 5 cards of the same color?

Standard 52-Card Deck

K ♠	K ♥	K ♦	K ♣
Q ♠	Q ♥	Q ♦	Q ♣
J ♠	J ♥	J ♦	J ♣
10 ♠	10 ♥	10 ♦	10 ♣
9 ♠	9 ♥	9 ♦	9 ♣
8 ♠	8 ♥	8 ♦	8 ♣
7 ♠	7 ♥	7 ♦	7 ♣
6 ♠	6 ♥	6 ♦	6 ♣
5 ♠	5 ♥	5 ♦	5 ♣
4 ♠	4 ♥	4 ♦	4 ♣
3 ♠	3 ♥	3 ♦	3 ♣
2 ♠	2 ♥	2 ♦	2 ♣
A ♠	A ♥	A ♦	A ♣

Solution

a. The number of ways to choose **5** cards from a deck of **52** cards is:

$${}_{52}C_5 = \frac{52!}{47! \cdot 5!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot \cancel{47!}}{\cancel{47!} \cdot 5!} = 2{,}598{,}960$$

b. For all 5 cards to be the same color, you need to choose **1** of the **2** colors and then **5** of the **26** cards in that color. So, the number of possible hands is:

$${}_2C_1 \cdot {}_{26}C_5 = \frac{2!}{1! \cdot 1!} \cdot \frac{26!}{21! \cdot 5!} = \frac{2}{1 \cdot 1} \cdot \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot \cancel{21!}}{\cancel{21!} \cdot 5!} = 131{,}560$$