PERMUTATIONS An ordering of n objects is a **permutation** of the objects. For instance, there are 6 permutations of the letters **A**, **B**, and **C**:

ABC **ACB** **BAC** **BCA** **CAB** **CBA**

You can use the fundamental counting principle to find the number of permutations of **A**, **B**, and **C**. There are **3** choices for the first letter. After the first letter has been chosen, **2** choices remain for the second letter. Finally, after the first two letters have been chosen, there is only **1** choice remaining for the final letter. So, the number of permutations is $3 \cdot 2 \cdot 1 = 6$.

DEFINE FACTORIALS

Zero factorial is defined as $0! = 1$.

The expression $3 \cdot 2 \cdot 1$ can also be written as 3!. The symbol ! is the **factorial** symbol, and 3! is read as "three factorial." In general, $n!$ is defined where n is a positive integer as follows:

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

The number of permutations of n distinct objects is $n!$.

EXAMPLE 4 Find the number of permutations

OLYMPICS Ten teams are competing in the final round of the Olympic four-person bobsledding competition.

a. In how many different ways can the bobsledding teams finish the competition? (Assume there are no ties.)

b. In how many different ways can 3 of the bobsledding teams finish first, second, and third to win the gold, silver, and bronze medals?

Solution

a. There are 10! different ways that the teams can finish the competition.

$$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3{,}628{,}800$$

b. Any of the **10** teams can finish first, then any of the remaining **9** teams can finish second, and finally any of the remaining **8** teams can finish third. So, the number of ways that the teams can win the medals is:

$$10 \cdot 9 \cdot 8 = 720$$

✓ GUIDED PRACTICE for Example 4

3. WHAT IF? In Example 4, how would the answers change if there were 12 bobsledding teams competing in the final round of the competition?

The answer to part (b) of Example 4 is called the number of permutations of 10 objects taken 3 at a time. It is denoted by ${}_{10}P_3$. Notice that this permutation can be computed using factorials:

$${}_{10}P_3 = 10 \cdot 9 \cdot 8 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10!}{7!} = \frac{10!}{(10 - 3)!}$$

This result is generalized at the top of the next page.