EXAMPLE 3 Use eccentricity to write a model

ASTRONOMY Pluto orbits the sun in an elliptical path with the center of the sun at one focus. The eccentricity of the orbit is $e = 0.249$ and the length of the major axis is about 79.0 astronomical units. Find an equation of Pluto's orbit. (Assume that the major axis is horizontal.)

Solution

The equation of the orbit has the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Using the length of the major axis, you know that $2a = 79.0$, or $a = 39.5$. You can use the eccentricity and the value of a to find the value of c, and then use the values of a and c to find b.

$e = \frac{c}{a}$, so $0.249 = \frac{c}{39.5}$, or $c \approx 9.84$

$c^2 = a^2 - b^2$, so $b = \sqrt{a^2 - c^2} = \sqrt{(39.5)^2 - (9.84)^2} \approx 38.3$

So, an equation for Pluto's orbit is $\frac{x^2}{(39.5)^2} + \frac{y^2}{(38.3)^2} = 1$, or $\frac{x^2}{1560} + \frac{y^2}{1470} = 1$, where x and y are measured in astronomical units.

PRACTICE

EXAMPLE 1 on p. 665 for Exs. 1–6

Find the eccentricity of the conic section.

1. $7(x - 3)^2 + 7(y + 7)^2 = 56$
2. $16(x + 1)^2 - 9(y - 5)^2 = 144$
3. $\frac{(x - 6)^2}{49} + \frac{(y - 5)^2}{64} = 1$
4. $\frac{(y - 4)^2}{100} - \frac{(x + 2)^2}{9} = 1$
5. $(x - 5)^2 = 10y$
6. $81(x + 4)^2 + (y - 9)^2 = 81$

EXAMPLE 2 on p. 665 for Exs. 7–12

Write an equation of the conic section.

7. Ellipse with vertices at $(-6, 4)$ and $(6, 4)$, and $e = 0.4$
8. Ellipse with foci at $(-4, 2)$ and $(-4, -2)$, and $e = 0.5$
9. Ellipse with center at $(0, 5)$, vertex at $(7, 5)$, and $e = 0.2$
10. Hyperbola with foci at $(4, -5)$ and $(4, 3)$, and $e = 2.5$
11. Hyperbola with vertices at $(1, -4)$ and $(7, -4)$, and $e = 1.8$
12. Hyperbola with center at $(-2, 3)$, focus at $(-5, 3)$, and $e = 4$

EXAMPLE 3 on p. 666 for Exs. 13–14

13. **ASTRONOMY** Nereid, a moon of Neptune, has the most eccentric orbit of any moon in the solar system. The eccentricity of the orbit is $e = 0.751$ and the length of the major axis is about 11.0 million kilometers. Find an equation of Nereid's orbit.

14. **SATELLITES** A communications satellite is in an elliptical orbit around Earth, whose center is one focus of the orbit. The eccentricity of the orbit is $e = 0.394$, and the satellite is 14,300 kilometers from Earth's center at the closest point in its orbit. What is the satellite's distance from Earth's center at the farthest point in its orbit?

15. **REASONING** *Explain* why the definition of eccentricity for ellipses and hyperbolas implies that $0 < e < 1$ for an ellipse and $e > 1$ for a hyperbola.