Extension
Use after Lesson 9.7

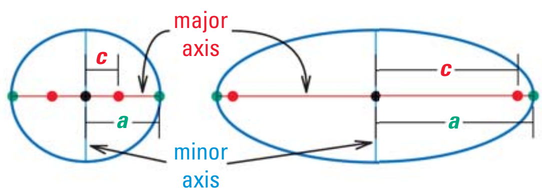
Determine Eccentricity of Conic Sections

TEKS a.1

GOAL Find and apply the eccentricity of a conic section.

Key Vocabulary
- eccentricity

In an ellipse that is nearly circular, the ratio $c:a$ is close to 0. In a more oval ellipse, $c:a$ is close to 1. This ratio is the **eccentricity** of the ellipse. Every conic has an eccentricity e associated with it.



KEY CONCEPT — *For Your Notebook*

Eccentricity of Conic Sections

The eccentricity of each conic section is defined below. For an ellipse or hyperbola, c is the distance from each focus to the center, and a is the distance from each vertex to the center.

Circle: $e = 0$

Parabola: $e = 1$

Ellipse: $e = \frac{c}{a}$, and $0 < e < 1$

Hyperbola: $e = \frac{c}{a}$, and $e > 1$

EXAMPLE 1 Find eccentricity

Find the eccentricity of the conic section represented by the equation.

a. $(x + 3)^2 + (y - 1)^2 = 25$

b. $\frac{(x+4)^2}{36} + \frac{(y-2)}{16} = 1$

Solution

a. Because this equation represents a circle, the eccentricity is $e = 0$.

b. This equation represents an ellipse with $a = \sqrt{36} = 6$, $b = \sqrt{16} = 4$, and $c = \sqrt{a^2 - b^2} = 2\sqrt{5}$. The eccentricity is $e = \frac{c}{a} = \frac{2\sqrt{5}}{6} \approx 0.745$.

EXAMPLE 2 Use eccentricity to write an equation

Write an equation of a hyperbola with center (−2, 6), vertex (6, 6), and $e = 2$.

Solution

Use the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. The vertex lies $6 - (-2) = 8$ units from the center, so $a = 8$. Because $e = \frac{c}{a} = 2$, you know that $\frac{c}{8} = 2$, or $c = 16$. So, $b^2 = c^2 - a^2 = 256 - 64 = 192$. The equation is $\frac{(x+2)^2}{64} - \frac{(y-6)^2}{192} = 1$.