EXAMPLE 3 Solve a quadratic system by elimination

Solve the system by elimination. $9x^2 + y^2 - 90x + 216 = 0$ **Equation 1**
$x^2 - y^2 - 16 = 0$ **Equation 2**

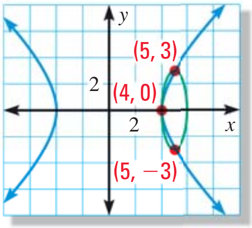
Solution

ANOTHER WAY
You can also solve by substitution: Solve Equation 2 for y^2, then substitute the result in Equation 1.

Add the equations to eliminate the y^2-term and obtain a quadratic equation in x.

$$\begin{array}{rl} 9x^2 + y^2 - 90x + 216 = 0 & \\ x^2 - y^2 \qquad - 16 = 0 & \\ \hline 10x^2 \qquad - 90x + 200 = 0 & \textbf{Add.} \\ x^2 - 9x + 20 = 0 & \textbf{Divide each side by 10.} \\ (x-4)(x-5) = 0 & \textbf{Factor.} \\ x = 4 \text{ or } x = 5 & \textbf{Zero product property} \end{array}$$

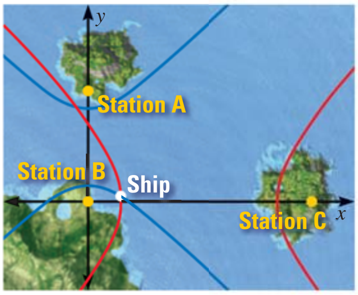
When $x = 4$, $y = 0$. When $x = 5$, $y = \pm 3$.



▶ The solutions are (4, 0), (5, 3), and (5, −3), as shown.

EXAMPLE 4 Solve a real-life quadratic system

NAVIGATION A ship uses LORAN (long-distance radio navigation) to find its position. Radio signals from stations A and B locate the ship on the blue hyperbola, and signals from stations B and C locate the ship on the red hyperbola. The equations of the hyperbolas are given below. Find the ship's position if it is east of the y-axis.



$x^2 - y^2 - 16x + 32 = 0$ **Equation 1**
$-x^2 + y^2 - 8y + 8 = 0$ **Equation 2**

Solution

STEP 1 **Add** the equations to eliminate the x^2- and y^2-terms.

$$\begin{array}{rl} x^2 - y^2 - 16x \qquad + 32 = 0 & \\ -x^2 + y^2 \qquad - 8y + 8 = 0 & \\ \hline -16x - 8y + 40 = 0 & \textbf{Add.} \\ y = -2x + 5 & \textbf{Solve for } \boldsymbol{y}\textbf{.} \end{array}$$

STEP 2 **Substitute** $-2x + 5$ for y in Equation 1 and solve for x.

$$\begin{array}{rl} x^2 - y^2 - 16x + 32 = 0 & \textbf{Equation 1} \\ x^2 - (-2x + 5)^2 - 16x + 32 = 0 & \textbf{Substitute for } \boldsymbol{y}\textbf{.} \\ 3x^2 - 4x - 7 = 0 & \textbf{Simplify.} \\ (x+1)(3x-7) = 0 & \textbf{Factor.} \\ x = -1 \text{ or } x = \frac{7}{3} & \textbf{Zero product property} \end{array}$$

STEP 3 **Substitute** for x in $y = -2x + 5$ to find the solutions $(-1, 7)$ and $\left(\frac{7}{3}, \frac{1}{3}\right)$.

▶ Because the ship is east of the y-axis, it is at $\left(\frac{7}{3}, \frac{1}{3}\right)$.