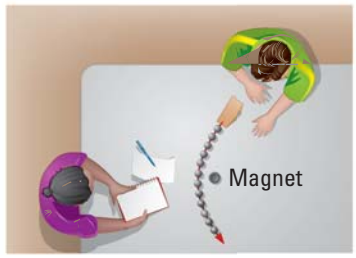
EXAMPLE 7 TAKS REASONING: Multi-Step Problem

PHYSICAL SCIENCE In a lab experiment, you record images of a steel ball rolling past a magnet. The equation $16x^2 - 9y^2 - 96x + 36y - 36 = 0$ models the ball's path.



- What is the shape of the path?
- Write an equation for the path in standard form.
- Graph the equation of the path.

Solution

STEP 1 **Identify** the shape. The equation is a general second-degree equation with $A = 16$, $B = 0$, and $C = -9$. Find the value of the discriminant.

$$B^2 - 4AC = 0^2 - 4(16)(-9) = 576$$

Because $B^2 - 4AC > 0$, the shape of the path is a hyperbola.

STEP 2 **Write** an equation. To write an equation of the hyperbola, complete the square in both x and y simultaneously.

$$16x^2 - 9y^2 - 96x + 36y - 36 = 0$$

$$(16x^2 - 96x) - (9y^2 - 36y) = 36$$

$$16(x^2 - 6x + \boxed{?}) - 9(y^2 - 4y + \boxed{?}) = 36 + 16(\boxed{?}) - 9(\boxed{?})$$

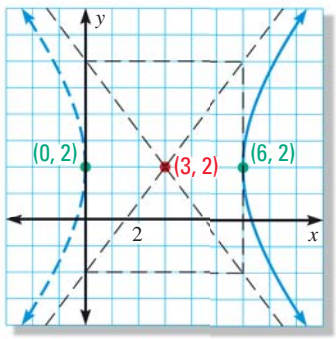
$$16(x^2 - 6x + \mathbf{9}) - 9(y^2 - 4y + \mathbf{4}) = 36 + 16(\mathbf{9}) - 9(\mathbf{4})$$

$$16(x - 3)^2 - 9(y - 2)^2 = 144$$

$$\frac{(x - 3)^2}{9} - \frac{(y - 2)^2}{16} = 1$$

AVOID ERRORS
To complete the square in two variables, you must add a quantity to or subtract a quantity from each side for *each* variable.

STEP 3 **Graph** the equation. From the equation, the transverse axis is horizontal, $\mathbf{(h, k) = (3, 2)}$, $a = \sqrt{9} = 3$, and $b = \sqrt{16} = 4$. The vertices are at $(3 \pm a, 2)$, or **(6, 2)** and **(0, 2)**.



Plot the center and vertices. Then draw a rectangle $2a = 6$ units wide and $2b = 8$ units high centered at **(3, 2)**, draw the asymptotes, and draw the hyperbola.

Notice that the path of the ball is modeled by just the right-hand branch of the hyperbola.

GUIDED PRACTICE for Examples 6 and 7

Classify the conic section and write its equation in standard form. Then graph the equation.

10. $x^2 + y^2 - 2x + 4y + 1 = 0$

11. $2x^2 + y^2 - 4x - 4 = 0$

12. $y^2 - 4y - 2x + 6 = 0$

13. $4x^2 - y^2 - 16x - 4y - 4 = 0$

14. **ASTRONOMY** An asteroid's path is modeled by $4x^2 - 6.25y^2 - 12x - 16 = 0$ where x and y are in astronomical units from the sun. Classify the path and write its equation in standard form. Then graph the equation.