



EXAMPLE 4 Write an equation of a translated ellipse

Write an equation of the ellipse with foci at (1, 2) and (7, 2) and co-vertices at (4, 0) and (4, 4).

Solution

STEP 1 Determine the form of the equation. First sketch the ellipse. The foci lie on the major axis, so the axis is horizontal. The equation has this form:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

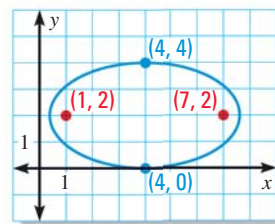
STEP 2 Identify h and k by finding the center, which is halfway between the foci (or the co-vertices).

$$(h, k) = \left(\frac{1+7}{2}, \frac{2+2}{2} \right) = (4, 2)$$

STEP 3 Find b , the distance between a co-vertex and the center (4, 2), and c , the distance between a focus and the center. Choose the co-vertex (4, 4) and the focus (1, 2): $b = |4 - 2| = 2$ and $c = |1 - 4| = 3$.

STEP 4 Find a . For an ellipse, $a^2 = b^2 + c^2 = 2^2 + 3^2 = 13$, so $a = \sqrt{13}$.

▶ The standard form of the equation is $\frac{(x - 4)^2}{13} + \frac{(y - 2)^2}{4} = 1$.



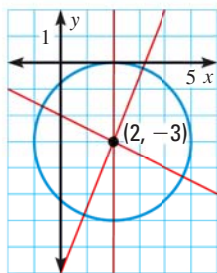
FIND DISTANCE

The co-vertices lie on a vertical line through the center and the foci lie on a horizontal line through the center, so you do not have to use the distance formula.

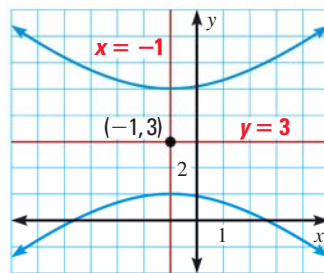
EXAMPLE 5 Identify symmetries of conic sections

Identify the line(s) of symmetry for each conic section in Examples 1–4.

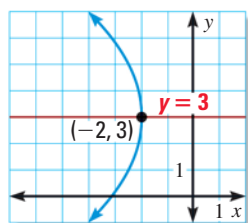
Solution



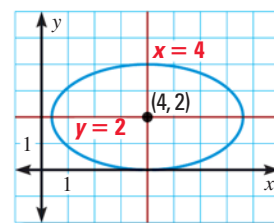
For the circle in Example 1, any line through the center (2, -3) is a line of symmetry.



For the hyperbola in Example 2, $x = -1$ and $y = 3$ are lines of symmetry.



For the parabola in Example 3, $y = 3$ is a line of symmetry.



For the ellipse in Example 4, $x = 4$ and $y = 2$ are lines of symmetry.