EXAMPLE 1 Graph an equation of a hyperbola

Graph $25y^2 - 4x^2 = 100$. Identify the vertices, foci, and asymptotes of the hyperbola.

Solution

STEP 1 **Rewrite** the equation in standard form.

$25y^2 - 4x^2 = 100$ **Write original equation.**

$\frac{25y^2}{100} - \frac{4x^2}{100} = \frac{100}{100}$ **Divide each side by 100.**

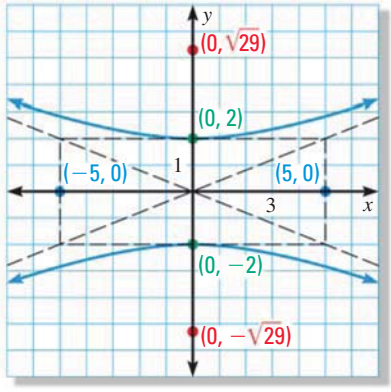
$\frac{y^2}{4} - \frac{x^2}{25} = 1$ **Simplify.**

STEP 2 **Identify** the vertices, foci, and asymptotes. Note that $a^2 = 4$ and $b^2 = 25$, so $a = 2$ and $b = 5$. The y^2-term is positive, so the transverse axis is vertical and the vertices are at $(0, \pm 2)$. Find the foci.

$c^2 = a^2 + b^2 = 2^2 + 5^2 = 29$, so $c = \sqrt{29}$

The foci are at $(0, \pm\sqrt{29}) \approx (0, \pm 5.4)$.

The asymptotes are $y = \pm\frac{a}{b}x$, or $y = \pm\frac{2}{5}x$.



STEP 3 **Draw** the hyperbola. First draw a rectangle centered at the origin that is $2a = 4$ units high and $2b = 10$ units wide. The asymptotes pass through opposite corners of the rectangle. Then, draw the hyperbola passing through the vertices and approaching the asymptotes.

Animated Algebra at classzone.com

SOLVE FOR *y*

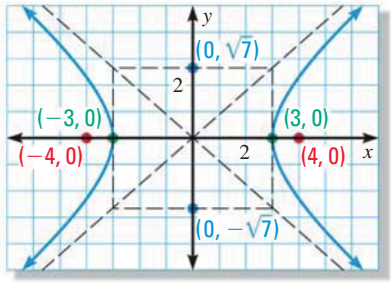
To plot points on the hyperbola, solve its equation for y to obtain $y = \pm 2\sqrt{1 + \frac{x^2}{25}}$. Then make a table of values.

EXAMPLE 2 Write an equation of a hyperbola

Write an equation of the hyperbola with foci at $(-4, 0)$ and $(4, 0)$ and vertices at $(-3, 0)$ and $(3, 0)$.

Solution

The foci and vertices lie on the x-axis equidistant from the origin, so the transverse axis is horizontal and the center is the origin. The foci are each 4 units from the center, so $c = 4$. The vertices are each 3 units from the center, so $a = 3$.



Because $c^2 = a^2 + b^2$, you have $b^2 = c^2 - a^2$. Find b^2.

$b^2 = c^2 - a^2 = 4^2 - 3^2 = 7$

Because the transverse axis is horizontal, the standard form of the equation is as follows:

$\frac{x^2}{3^2} - \frac{y^2}{7} = 1$ **Substitute 3 for a and 7 for b^2.**

$\frac{x^2}{9} - \frac{y^2}{7} = 1$ **Simplify.**