PROBLEM SOLVING WORKSHOP
LESSON 9.4

Using ALTERNATIVE METHODS

TEKS a.4, a.5, a.6, 2A.5.C

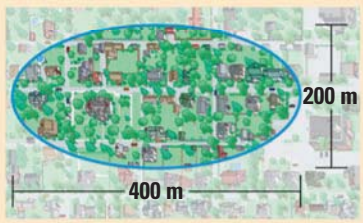
Another Way to Solve Example 3, page 636

MULTIPLE REPRESENTATIONS In the second part of Example 3 on page 636, you found the area of an ellipse using a formula. You can also approximate the area of an ellipse by summing the areas of rectangles.

PROBLEM

LIGHTNING When lightning strikes, an elliptical region where the strike most likely hit can often be identified. Suppose it is determined that there is a 50% chance that a lightning strike hit within the elliptical region shown in the diagram.

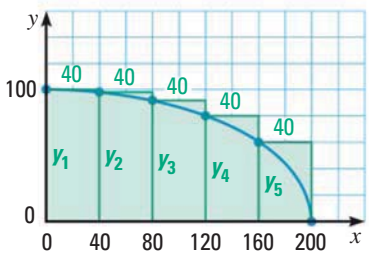
- Write an equation of the ellipse.
- Find the area of the elliptical region.



METHOD

Summing Rectangles As you saw on page 636, the ellipse has the equation $\frac{x^2}{200^2} + \frac{y^2}{100^2} = 1$. Approximate the area of the ellipse as follows.

STEP 1 **Graph** the first-quadrant portion of the ellipse. Then draw rectangles of width 40 and height equal to the y-value of the ellipse at the rectangle's left edge. The first rectangle's height is $y_1 = 100$. To find the other y-values, solve for y to obtain $y = \sqrt{100^2 - \frac{x^2}{4}}$. Use a calculator to get $y_2 \approx 98.0$, $y_3 \approx 91.7$, $y_4 = 80$, and $y_5 = 60$.



STEP 2 **Calculate** the total area A of the rectangles.

$$A \approx 40(100) + 40(98.0) + 40(91.7) + 40(80) + 40(60) = 17{,}188 \text{ m}^2$$

STEP 3 **Multiply** the total area of the rectangles by 4 to obtain an estimate of $4(17{,}188) \approx 68{,}800$ square meters for the area of the ellipse.

PRACTICE

1. Solve the problem above using rectangles of width 20. Is this estimate better or worse than the estimate above? *Explain*.
2. **REASONING** *Explain* using your results from Exercise 1 how to obtain a closer and closer approximation of the ellipse's area.
3. **WHAT IF?** Suppose that the ellipse in the problem had a horizontal major axis of 250 meters and a minor axis of 200 meters.
   a. Write an equation of the ellipse.
   b. Use the method above to approximate the area of the ellipse.