EXAMPLE 2 Write an equation of a circle

The point (2, −5) lies on a circle whose center is the origin. Write the standard form of the equation of the circle.

Solution

Because the point (2, −5) lies on the circle, the circle's radius r must be the distance between the center (0, 0) and (2, −5). Use the distance formula.

$r = \sqrt{(2-0)^2 + (-5-0)^2} = \sqrt{4 + 25} = \sqrt{29}$ **The radius is $\sqrt{29}$.**

Use the standard form with $r = \sqrt{29}$ to write an equation of the circle.

$x^2 + y^2 = r^2$ **Standard form**

$x^2 + y^2 = (\sqrt{29})^2$ **Substitute $\sqrt{29}$ for r.**

$x^2 + y^2 = 29$ **Simplify.**

TAKS EXAMPLE 3 TAKS PRACTICE: Multiple Choice

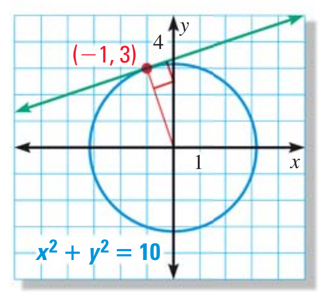
ELIMINATE CHOICES
In Example 3, you can eliminate choice D because a quick sketch of the circle shows that the slope of the tangent line at (−1, 3) must be positive.

What is an equation of the line tangent to the circle $x^2 + y^2 = 10$ at (−1, 3)?

(A) $y = 3x + 4$ (B) $y = 3x + \frac{8}{3}$ (C) $y = \frac{1}{3}x + \frac{10}{3}$ (D) $y = -\frac{1}{3}x + \frac{8}{3}$

Solution

From geometry, a line tangent to a circle is perpendicular to the radius at the point of tangency. The radius with endpoint (−1, 3) has slope $m = \frac{3-0}{-1-0} = -3$, so the slope of the tangent line at (−1, 3) is the negative reciprocal of −3, or $\frac{1}{3}$.



An equation of the tangent line is as follows:

$y - 3 = \frac{1}{3}(x - (-1))$ **Point-slope form**

$y - 3 = \frac{1}{3}x + \frac{1}{3}$ **Distributive property**

$y = \frac{1}{3}x + \frac{10}{3}$ **Solve for y.**

▸ The correct answer is C. (A) (B) (C) (D)

✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the equation. Identify the radius of the circle.

1. $x^2 + y^2 = 9$
2. $y^2 = -x^2 + 49$
3. $x^2 - 18 = -y^2$
4. Write the standard form of the equation of the circle that passes through (5, −1) and whose center is the origin.
5. Write an equation of the line tangent to the circle $x^2 + y^2 = 37$ at (6, 1).