EXAMPLE 3 Solve a rational inequality algebraically

Solve $\frac{6}{x-2} \geq -4$ algebraically.

Solution

STEP 1 **Rewrite** the inequality so that one side is 0. Then write the other side as a simplified rational expression.

AVOID ERRORS
Do not multiply each side of an inequality by an expression involving *x* if the expression can take on both positive and negative values.

$$\frac{6}{x-2} \geq -4 \quad \text{Write original inequality.}$$

$$\frac{6}{x-2} + 4 \geq 0 \quad \text{Add 4 to each side.}$$

$$\frac{6 + 4(x-2)}{x-2} \geq 0 \quad \text{Write left side as a single fraction.}$$

$$\frac{4x-2}{x-2} \geq 0 \quad \text{Simplify.}$$

STEP 2 **Identify** the *critical x-values*, which are the *x*-values that make the numerator or denominator equal to 0.

Numerator equal to 0:

$4x - 2 = 0$

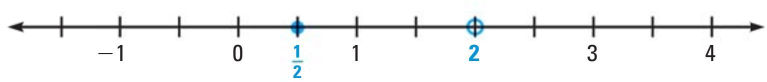
$x = \frac{1}{2}$

Denominator equal to 0:

$x - 2 = 0$

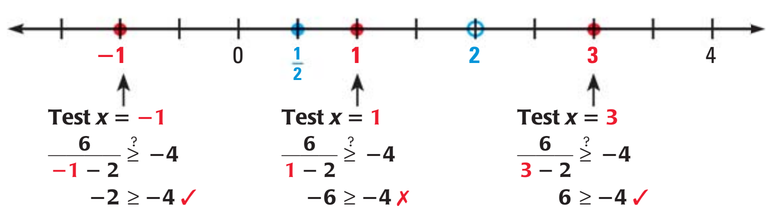
$x = 2$

So, the critical *x*-values are $x = \frac{1}{2}$ and $x = 2$.

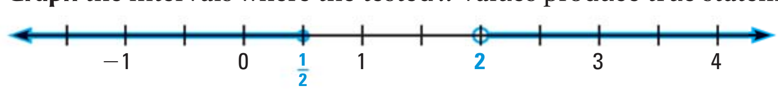


The critical *x*-values divide the number line into three intervals. Note that $x = \frac{1}{2}$ will be included in the solution, but $x = 2$ will not because it results in division by zero.

STEP 3 **Test** an *x*-value in each interval to see if it satisfies the original inequality. If it does, *every* *x*-value in the interval will satisfy the inequality. If it does not, *no* *x*-value in the interval will satisfy the inequality.



STEP 4 **Graph** the intervals where the tested *x*-values produce true statements.



STEP 5 **Write** inequalities to describe the solution.

▸ The solution is $x \leq \frac{1}{2}$ or $x > 2$.