EXTRANEOUS SOLUTIONS When solving a rational equation, you may obtain solutions that are extraneous. Be sure to check for extraneous solutions by substituting back into the original equation.

EXAMPLE 5 Check for extraneous solutions

Solve: $\dfrac{6}{x-3} = \dfrac{8x^2}{x^2-9} - \dfrac{4x}{x+3}$

Solution

Write each denominator in factored form. The LCD is $(x+3)(x-3)$.

$$\frac{6}{x-3} = \frac{8x^2}{(x+3)(x-3)} - \frac{4x}{x+3}$$

$$(x+3)(x-3) \cdot \frac{6}{x-3} = (x+3)(x-3) \cdot \frac{8x^2}{(x+3)(x-3)} - (x+3)(x-3) \cdot \frac{4x}{x+3}$$

$$6(x+3) = 8x^2 - 4x(x-3)$$

$$6x + 18 = 8x^2 - 4x^2 + 12x$$

$$0 = 4x^2 + 6x - 18$$

$$0 = 2x^2 + 3x - 9$$

$$0 = (2x-3)(x+3)$$

$$2x - 3 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -3$$

REVIEW EXTRANEOUS SOLUTIONS
For help with extraneous solutions, see p. 51.

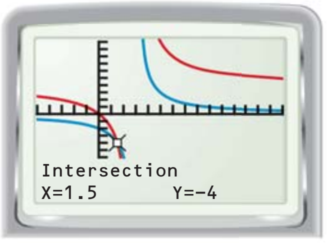
You can use algebra or a graph to check whether either of the two solutions is extraneous.

Algebra The solution $\frac{3}{2}$ checks, but the apparent solution -3 is extraneous, because substituting it in the equation results in division by zero, which is undefined.

$$\frac{6}{-3-3} \neq \frac{8(-3)^2}{(-3)^2-9} - \frac{4(-3)}{-3+3}$$

Division by zero is undefined

Graph Graph $y = \dfrac{6}{x-3}$ and $y = \dfrac{8x^2}{x^2-9} - \dfrac{4x}{x+3}$.



The graphs intersect when $x = \frac{3}{2}$, but not when $x = -3$.

▶ The solution is $\frac{3}{2}$.

✓ GUIDED PRACTICE for Examples 3, 4, and 5

Solve the equation by using the LCD. Check for extraneous solutions.

5. $\dfrac{7}{2} + \dfrac{3}{x} = 3$

6. $\dfrac{2}{x} + \dfrac{4}{3} = 2$

7. $\dfrac{3}{7} + \dfrac{8}{x} = 1$

8. $\dfrac{3}{2} + \dfrac{4}{x-1} = \dfrac{x+1}{x-1}$

9. $\dfrac{3x}{x+1} - \dfrac{5}{2x} = \dfrac{3}{2x}$

10. $\dfrac{5x}{x-2} = 7 + \dfrac{10}{x-2}$