KEY CONCEPT *For Your Notebook*

Adding or Subtracting with Unlike Denominators

To add (or subtract) two rational expressions with *unlike* denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add (or subtract).

Let a, b, c, and d be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

Subtraction

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

You can always find a common denominator of two rational expressions by multiplying their denominators, as shown above. However, if you use the least common denominator (LCD), which is the least common multiple (LCM) of the denominators, you may have less simplifying to do.

EXAMPLE 2 Find a least common multiple (LCM)

Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

Solution

STEP 1 **Factor** each polynomial. Write numerical factors as products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = (2^2)(x + 2)(x - 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = (2)(3)(x - 2)^2$$

STEP 2 **Form** the LCM by writing each factor to the highest power it occurs in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

EXAMPLE 3 Add with unlike denominators

Add: $\dfrac{7}{9x^2} + \dfrac{x}{3x^2 + 3x}$

REVIEW LCDS
For help with finding least common denominators, see p. 986.

Solution

To find the LCD, factor each denominator and write each factor to the highest power it occurs. Note that $9x^2 = 3^2x^2$ and $3x^2 + 3x = 3x(x + 1)$, so the LCD is $3^2x^2(x + 1) = 9x^2(x + 1)$.

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7}{9x^2} + \frac{x}{3x(x + 1)}$$ Factor second denominator.

$$= \frac{7}{9x^2} \cdot \frac{x + 1}{x + 1} + \frac{x}{3x(x + 1)} \cdot \frac{3x}{3x}$$ LCD is $9x^2(x + 1)$.

$$= \frac{7x + 7}{9x^2(x + 1)} + \frac{3x^2}{9x^2(x + 1)}$$ Multiply.

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)}$$ Add numerators.