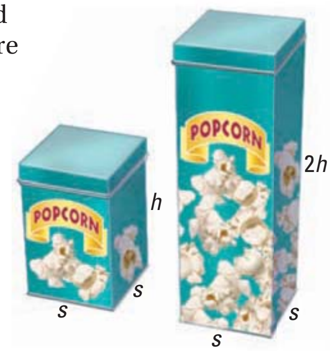
EFFICIENCY Manufacturers often package their products in a way that uses the least amount of packaging material. One measure of the efficiency of a package is the ratio of its surface area to its volume. The smaller the ratio, the more efficient the packaging.

TAKS EXAMPLE 2 TAKS REASONING: Multi-Step Problem

PACKAGING A company makes a tin to hold flavored popcorn. The tin is a rectangular prism with a square base. The company is designing a new tin with the same base and twice the height of the old tin.

- Find the surface area and volume of each tin.
- Calculate the ratio of surface area to volume for each tin.
- What do the ratios tell you about the efficiencies of the two tins?



Solution

	Old tin	New tin	
STEP 1	$S = 2s^2 + 4sh$	$S = 2s^2 + 4s(2h)$	**Find surface area, *S*.**
		$= 2s^2 + 8sh$	
	$V = s^2h$	$V = s^2(2h)$	**Find volume, *V*.**
		$= 2s^2h$	
STEP 2	$\frac{S}{V} = \frac{2s^2 + 4sh}{s^2h}$	$\frac{S}{V} = \frac{2s^2 + 8sh}{2s^2h}$	**Write ratio of *S* to *V*.**
	$= \frac{\cancel{s}(2s + 4h)}{\cancel{s}(sh)}$	$= \frac{\cancel{2s}(s + 4h)}{\cancel{2s}(sh)}$	**Divide out common factor.**
	$= \frac{2s + 4h}{sh}$	$= \frac{s + 4h}{sh}$	**Simplified form**

STEP 3 $\frac{2s + 4h}{sh} > \frac{s + 4h}{sh}$ because the left side of the inequality has a greater numerator than the right side and both have the same (positive) denominator. The ratio of surface area to volume is *greater* for the old tin than for the new tin. So, the old tin is *less* efficient than the new tin.

✓ GUIDED PRACTICE for Examples 1 and 2

Simplify the expression, if possible.

1. $\frac{2(x + 1)}{(x + 1)(x + 3)}$

2. $\frac{40x + 20}{10x + 30}$

3. $\frac{4}{x(x + 2)}$

4. $\frac{x + 4}{x^2 - 16}$

5. $\frac{x^2 - 2x - 3}{x^2 - x - 6}$

6. $\frac{2x^2 + 10x}{3x^2 + 16x + 5}$

7. **WHAT IF?** In Example 2, suppose the new popcorn tin is the same height as the old tin but has a base with sides twice as long. What is the ratio of surface area to volume for this tin?