8.4 Multiply and Divide Rational Expressions

Before	You graphed rational functions.
Now	You will multiply and divide rational expressions.
Why?	So you can compare the efficiencies of two designs, as in Ex. 51.

Key Vocabulary
- simplified form of a rational expression
- reciprocal, *p. 4*

A rational expression is in **simplified form** if its numerator and denominator have no common factors (other than ±1). To simplify a rational expression, apply the following property.

KEY CONCEPT *For Your Notebook*

Simplifying Rational Expressions

Let a, b, and c be expressions with $b \neq 0$ and $c \neq 0$. Then the following property applies.

Property $\dfrac{ac}{bc} = \dfrac{a}{b}$ **Divide out the common factor c.**

Examples $\dfrac{15}{65} = \dfrac{3 \cdot 5}{13 \cdot 5} = \dfrac{3}{13}$ **Divide out the common factor 5.**

$\dfrac{4(x+3)}{(x-5)(x+3)} = \dfrac{4}{x-5}$ **Divide out the common factor $x + 3$.**

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any factors that are common to both the numerator and denominator. Here is an example:

$$\frac{x^2 + 7x}{x^2} = \frac{x(x+7)}{x \cdot x} = \frac{x+7}{x}$$

Notice that you can divide out common factors in the second expression above. However, you cannot divide out like terms in the third expression.

EXAMPLE 1 **Simplify a rational expression**

Simplify: $\dfrac{x^2 - 2x - 15}{x^2 - 9}$

AVOID ERRORS
Do not divide out variable terms that are not factors.
$$\frac{x-5}{x-3} \neq \frac{-5}{-3}$$

$\dfrac{x^2 - 2x - 15}{x^2 - 9} = \dfrac{(x+3)(x-5)}{(x+3)(x-3)}$ **Factor numerator and denominator.**

$= \dfrac{(x+3)(x-5)}{(x+3)(x-3)}$ **Divide out common factor.**

$= \dfrac{x-5}{x-3}$ **Simplified form**