GUIDED PRACTICE for Examples 1, 2, and 3

Graph the function.

1. $y = \dfrac{4}{x^2 + 2}$ **2.** $y = \dfrac{3x^2}{x^2 - 1}$ **3.** $f(x) = \dfrac{x^2 - 5}{x^2 + 1}$ **4.** $y = \dfrac{x^2 - 2x - 3}{x - 4}$

EXAMPLE 4 TAKS REASONING: Multi-Step Problem

MANUFACTURING A food manufacturer wants to find the most efficient packaging for a can of soup with a volume of 342 cubic centimeters. Find the dimensions of the can that has this volume and uses the least amount of material possible.

Solution

STEP 1 **Write** an equation that gives the height h of the soup can in terms of its radius r. Use the formula for the volume of a cylinder and the fact that the soup can's volume is 342 cubic centimeters.

$V = \pi r^2 h$ Formula for volume of cylinder

$342 = \pi r^2 h$ Substitute 342 for V.

$\dfrac{342}{\pi r^2} = h$ Solve for h.

STEP 2 **Write** a function that gives the surface area S of the soup can in terms of only its radius r.

$S = 2\pi r^2 + 2\pi r h$ Formula for surface area of cylinder

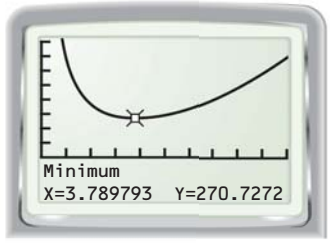
$= 2\pi r^2 + 2\pi r\left(\dfrac{342}{\pi r^2}\right)$ Substitute $\dfrac{342}{\pi r^2}$ for h.

$= 2\pi r^2 + \dfrac{684}{r}$ Simplify.

INTERPRET FUNCTIONS
The function for the surface area is a rational function because it can be written as a quotient of polynomials:

$$S = \frac{2\pi r^3 + 684}{r}$$

STEP 3 **Graph** the function for the surface area S using a graphing calculator. Then use the *minimum* feature to find the minimum value of S.



You get a minimum value of about 271, which occurs when $r \approx 3.79$ and $h \approx \dfrac{342}{\pi(3.79)^2} \approx 7.58$.

▶ So, the soup can using the least amount of material has a radius of about 3.79 centimeters and a height of about 7.58 centimeters. Notice that the height and the diameter are equal for this can.

GUIDED PRACTICE for Example 4

5. WHAT IF? In Example 4, suppose the manufacturer wants to find the most efficient packaging for a soup can with a volume of 544 cubic centimeters. Find the dimensions of this can.