8.3 Graph General Rational Functions

TEKS 2A.10.A, 2A.10.B, 2A.10.C, 2A.10.F

Before You graphed rational functions involving linear polynomials.

Now You will graph rational functions with higher-degree polynomials.

Why? So you can solve problems about altitude, as in Ex. 35.

Key Vocabulary
- **end behavior,** *p. 339*
- **asymptote,** *p. 478*
- **rational function,** *p. 558*

KEY CONCEPT *For Your Notebook*

Graphs of Rational Functions

Let $p(x)$ and $q(x)$ be polynomials with no common factors other than ± 1. The graph of the following rational function has the characteristics listed below.

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$$

1. The x-intercepts of the graph of f are the real zeros of $p(x)$.
2. The graph of f has a vertical asymptote at each real zero of $q(x)$.
3. The graph of f has at most one horizontal asymptote, which is determined by the degrees m and n of $p(x)$ and $q(x)$.

$m < n$	The line $y = 0$ is a horizontal asymptote.
$m = n$	The line $y = \frac{a_m}{b_n}$ is a horizontal asymptote.
$m > n$	The graph has no horizontal asymptote. The graph's end behavior is the same as the graph of $y = \frac{a_m}{b_n}x^{m-n}$.

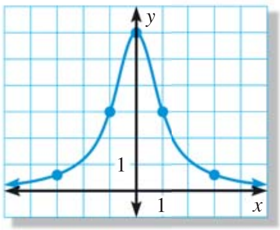
EXAMPLE 1 Graph a rational function ($m < n$)

Graph $y = \frac{6}{x^2 + 1}$. State the domain and range.

Solution

The numerator has no zeros, so there is no x-intercept. The denominator has no real zeros, so there is no vertical asymptote.

The degree of the numerator, 0, is less than the degree of the denominator, 2. So, the line $y = 0$ (the x-axis) is a horizontal asymptote.



The graph passes through the points $(-3, 0.6)$, $(-1, 3)$, $(0, 6)$, $(1, 3)$, and $(3, 0.6)$. The domain is all real numbers, and the range is $0 < y \le 6$.