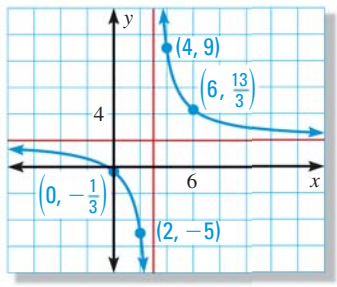
EXAMPLE 3 Graph a rational function of the form $y = \frac{ax + b}{cx + d}$

Graph $y = \frac{2x + 1}{x - 3}$. State the domain and range.

Solution



STEP 1 **Draw** the asymptotes. Solve $x - 3 = 0$ for x to find the vertical asymptote $x = 3$. The horizontal asymptote is the line $y = \frac{a}{c} = \frac{2}{1} = 2$.

STEP 2 **Plot** points to the left of the vertical asymptote, such as $(2, -5)$ and $\left(0, -\frac{1}{3}\right)$, and points to the right, such as $(4, 9)$ and $\left(6, \frac{13}{3}\right)$.

STEP 3 **Draw** the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

▸ The domain is all real numbers except 3. The range is all real numbers except 2.

EXAMPLE 4 TAKS REASONING: Multi-Step Problem

3-D MODELING A 3-D printer builds up layers of material to make three-dimensional models. Each deposited layer bonds to the layer below it. A car company decides to make small display models of its vehicles using a 3-D printer. The printer costs $24,000. The material for each model costs $300.

- Write an equation that gives the average cost per model as a function of the number of models printed.
- Graph the function. Use the graph to estimate how many models must be printed for the average cost per model to fall to $700.
- What happens to the average cost as more models are printed?

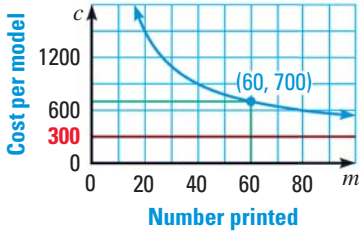
Solution

STEP 1 **Write** a function. Let c be the average cost and m be the number of models printed.

$$c = \frac{\text{Unit cost} \cdot \text{Number printed} + \text{Cost of printer}}{\text{Number printed}} = \frac{300m + 24{,}000}{m}$$

DRAW GRAPHS
Because the number of models and average cost cannot be negative, graph only the branch of the hyperbola that lies in the first quadrant.

STEP 2 **Graph** the function. The asymptotes are the lines $m = 0$ and $c = 300$. The average cost falls to $700 per model after 60 models are printed.



STEP 3 **Interpret** the graph. As more models are printed, the average cost per model approaches $300.