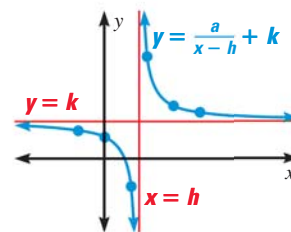


## Graphing Translations of Simple Rational Functions

To graph a rational function of the form  $y = \frac{a}{x-h} + k$ , follow these steps:

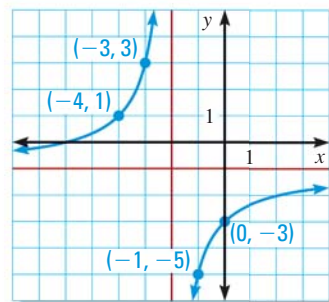
- STEP 1** Draw the asymptotes  $x = h$  and  $y = k$ .
- STEP 2** Plot points to the left and to the right of the vertical asymptote.
- STEP 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.


**EXAMPLE 2** Graph a rational function of the form  $y = \frac{a}{x-h} + k$ 

Graph  $y = \frac{-4}{x+2} - 1$ . State the domain and range.

**Solution**

- STEP 1** Draw the asymptotes  $x = -2$  and  $y = -1$ .
- STEP 2** Plot points to the left of the vertical asymptote, such as  $(-3, 3)$  and  $(-4, 1)$ , and points to the right, such as  $(-1, -5)$  and  $(0, -3)$ .
- STEP 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



The domain is all real numbers except  $-2$ , and the range is all real numbers except  $-1$ .

at classzone.com

**INTERPRET TRANSFORMATIONS**

The graph of  $y = \frac{-4}{x+2} - 1$  is the graph of  $y = \frac{-4}{x}$  translated left 2 units and down 1 unit.

**GUIDED PRACTICE** for Examples 1 and 2

Graph the function. State the domain and range.

1.  $f(x) = \frac{-4}{x}$

2.  $y = \frac{8}{x} - 5$

3.  $y = \frac{1}{x-3} + 2$

**OTHER RATIONAL FUNCTIONS** All rational functions of the form  $y = \frac{ax+b}{cx+d}$  also have graphs that are hyperbolas.

- The vertical asymptote of the graph is the line  $x = -\frac{d}{c}$ , because the function is undefined when the denominator  $cx + d$  is zero.
- The horizontal asymptote is the line  $y = \frac{a}{c}$ .