8.2 Graph Simple Rational Functions



You graphed polynomial functions. You will graph rational functions. So you can find average monthly costs, as in Ex. 38.



Key Vocabulary

rational function

- domain, *p.* 72
- range, p. 72
- asymptote, *p.* 478

A **rational function** has the form $f(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomials and $q(x) \neq 0$. The inverse variation function $f(x) = \frac{a}{x}$ is a rational function. The graph of this function when a = 1 is shown below.

KEY CONCEPT

For Your Notebook

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called *branches*. The domain and range are all nonzero real numbers.

Any function of the form $g(x) = \frac{a}{x}$ ($a \neq 0$) has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.



EXAMPLE 1 Graph a rational function of the form $y = \frac{a}{x}$

Graph the function $y = \frac{6}{x}$. Compare the graph with the graph of $y = \frac{1}{x}$.

Solution

- **STEP 1** Draw the asymptotes x = 0 and y = 0.
- **STEP 2** Plot points to the left and to the right of the vertical asymptote, such as (-3, -2), (-2, -3), (2, 3), and (3, 2).
- *STEP 3* **Draw** the branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



The graph of $y = \frac{6}{x}$ lies farther from the axes than the graph of $y = \frac{1}{x}$.

Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

INTERPRET TRANSFORMATIONS The graph of $y = \frac{6}{x}$ is a

vertical stretch of the graph of $y = \frac{1}{x}$ by a factor of 6.