Solve Exponential and Logarithmic Inequalities

TEKS *2A.11.E, 2A.11.F*

GOAL Solve exponential and logarithmic inequalities using tables and graphs.

In the Problem Solving Workshop on pages 523–525, you learned how to solve exponential and logarithmic equations using tables and graphs. You can use these same methods to solve exponential and logarithmic inequalities.

EXAMPLE 1 Solve an exponential inequality

CARS Your family purchases a new car for $20,000. Its value decreases by 15% each year. During what interval of time does the car's value exceed $10,000?

Solution

Let y represent the value of the car (in dollars) x years after it is purchased. A function relating x and y is $y = 20,000(1 - 0.15)^x$, or $y = 20,000(0.85)^x$. To find the values of x for which $y > 10,000$, solve the inequality $20,000(0.85)^x > 10,000$.

METHOD 1 Use a table

STEP 1 **Enter** the function $y = 20,000(0.85)^x$ into a graphing calculator. Set the starting x-value of the table to 0 and the step value to 0.1.

```
TABLE SETUP
 TblStart=0
 △Tbl=0.1
Indpnt: Auto Ask
Depend: Auto Ask
```

STEP 2 **Use** the *table* feature to create a table of values. Scrolling through the table shows that $y > 10,000$ when $0 \le x \le 4.2$.

```
  X    │    Y₁
 4     │  10440
 4.1   │  10272
 4.2   │  10106
 4.3   │  9943.3
 4.4   │  9783
X=4.2
```
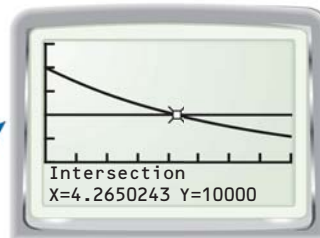
▶ The car value exceeds $10,000 for about the first 4.2 years after it is purchased.
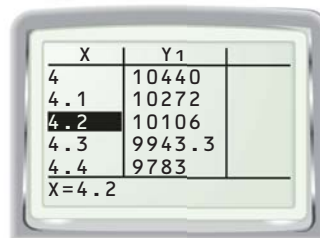
To check the solution's reasonableness, note that $y \approx 10,440$ when $x = 4$ and $y \approx 8874$ when $x = 5$. So, $4 < x < 5$, which agrees with the solution obtained above.

METHOD 2 Use a graph

Graph $y = 20,000(0.85)^x$ and $y = 10,000$ in the same viewing window. Set the viewing window to show $0 \le x \le 8$ and $0 \le y \le 25,000$. Using the *intersect* feature, you can determine that the graphs intersect when $x \approx 4.27$.

```
Intersection
X=4.2650243 Y=10000
```

> The graph of $y = 20,000(0.85)^x$ is above the graph of $y = 10,000$ when $0 \le x < 4.27$.

▶ The car value exceeds $10,000 for about the first 4.27 years after it is purchased.