EXTRANEOUS SOLUTIONS Because the domain of a logarithmic function generally does not include all real numbers, be sure to check for extraneous solutions of logarithmic equations. You can do this algebraically or graphically.

EXAMPLE 6 TAKS PRACTICE: Multiple Choice

What is (are) the solution(s) of $\log 8x + \log (x - 20) = 3$?

Ⓐ $-5, 25$ Ⓑ 5 Ⓒ 25 Ⓓ $5, 25$

ELIMINATE CHOICES
Instead of solving the equation in Example 6 directly, you can substitute each possible answer into the equation to see whether it is a solution.

Solution

$\log 8x + \log (x - 20) = 3$ — **Write original equation.**

$\log [8x(x - 20)] = 3$ — **Product property of logarithms**

$10^{\log [8x(x - 20)]} = 10^3$ — **Exponentiate each side using base 10.**

$8x(x - 20) = 1000$ — $b^{\log_b x} = x$

$8x^2 - 160x = 1000$ — **Distributive property**

$8x^2 - 160x - 1000 = 0$ — **Write in standard form.**

$x^2 - 20x - 125 = 0$ — **Divide each side by 8.**

$(x - 25)(x + 5) = 0$ — **Factor.**

$x = 25$ or $x = -5$ — **Zero product property**

CHECK Check the apparent solutions 25 and -5 using algebra or a graph.

Algebra Substitute 25 and -5 for x in the original equation.

$\log 8x + \log (x - 20) = 3$

$\log (8 \cdot 25) + \log (25 - 20) \stackrel{?}{=} 3$

$\log 200 + \log 5 \stackrel{?}{=} 3$

$\log 1000 \stackrel{?}{=} 3$

$3 = 3$ ✓

So, 25 is a solution.

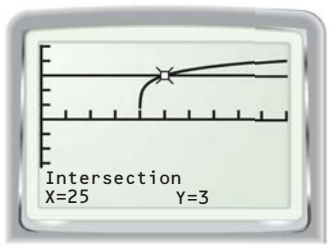
$\log 8x + \log (x - 20) = 3$

$\log [8(-5)] + \log (-5 - 20) \stackrel{?}{=} 3$

$\log (-40) + \log (-25) \stackrel{?}{=} 3$

Because $\log (-40)$ and $\log (-25)$ are not defined, -5 is *not* a solution.

Graph Graph $y = \log 8x + \log (x - 20)$ and $y = 3$ in the same coordinate plane. The graphs intersect only once, when $x = 25$. So, 25 is the only solution.



▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 4, 5, and 6

Solve the equation. Check for extraneous solutions.

7. $\ln (7x - 4) = \ln (2x + 11)$

8. $\log_2 (x - 6) = 5$

9. $\log 5x + \log (x - 1) = 2$

10. $\log_4 (x + 12) + \log_4 x = 3$