SOLVING LOGARITHMIC EQUATIONS **Logarithmic equations** are equations that involve logarithms of variable expressions. You can use the following property to solve some types of logarithmic equations.

KEY CONCEPT *For Your Notebook*

Property of Equality for Logarithmic Equations

Algebra If b, x, and y are positive numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Example If $\log_2 x = \log_2 7$, then $x = 7$. If $x = 7$, then $\log_2 x = \log_2 7$.

EXAMPLE 4 Solve a logarithmic equation

Solve $\log_5 (4x - 7) = \log_5 (x + 5)$.

$\log_5 (4x - 7) = \log_5 (x + 5)$ — Write original equation.

$4x - 7 = x + 5$ — Property of equality for logarithmic equations

$3x - 7 = 5$ — Subtract x from each side.

$3x = 12$ — Add 7 to each side.

$x = 4$ — Divide each side by 3.

▶ The solution is 4.

CHECK Check the solution by substituting it into the original equation.

$\log_5 (4x - 7) = \log_5 (x + 5)$ — Write original equation.

$\log_5 (4 \cdot 4 - 7) \stackrel{?}{=} \log_5 (4 + 5)$ — Substitute 4 for x.

$\log_5 9 = \log_5 9$ ✓ — Solution checks.

EXPONENTIATING TO SOLVE EQUATIONS The property of equality for exponential equations on page 515 implies that if you are given an equation $x = y$, then you can *exponentiate* each side to obtain an equation of the form $b^x = b^y$. This technique is useful for solving some logarithmic equations.

EXAMPLE 5 Exponentiate each side of an equation

Solve $\log_4 (5x - 1) = 3$.

$\log_4 (5x - 1) = 3$ — Write original equation.

$4^{\log_4 (5x - 1)} = 4^3$ — Exponentiate each side using base 4.

$5x - 1 = 64$ — $b^{\log_b x} = x$

$5x = 65$ — Add 1 to each side.

$x = 13$ — Divide each side by 5.

▶ The solution is 13.

CHECK $\log_4 (5x - 1) = \log_4 (5 \cdot 13 - 1) = \log_4 64$

Because $4^3 = 64$, $\log_4 64 = 3$. ✓