

When it is not convenient to write each side of an exponential equation using the same base, you can solve the equation by taking a logarithm of each side.

EXAMPLE 2 Take a logarithm of each side

ANOTHER WAY

For an alternative method for solving the problem in Example 2, turn to page 523 for the **Problem Solving Workshop**.

Solve $4^x = 11$.

$$4^x = 11$$

Write original equation.

$$\log_4 4^x = \log_4 11$$

Take \log_4 of each side.

$$x = \log_4 11$$

$\log_b b^x = x$

$$x = \frac{\log 11}{\log 4}$$

Change-of-base formula

$$x \approx 1.73$$

Use a calculator.

► The solution is about 1.73. Check this in the original equation.

NEWTON'S LAW OF COOLING An important application of exponential equations is *Newton's law of cooling*. This law states that for a cooling substance with initial temperature T_0 , the temperature T after t minutes can be modeled by

$$T = (T_0 - T_R)e^{-rt} + T_R$$

where T_R is the surrounding temperature and r is the substance's cooling rate.

EXAMPLE 3 Use an exponential model

CARS You are driving on a hot day when your car overheats and stops running. It overheats at 280°F and can be driven again at 230°F . If $r = 0.0048$ and it is 80°F outside, how long (in minutes) do you have to wait until you can continue driving?



Solution

$$T = (T_0 - T_R)e^{-rt} + T_R$$

Newton's law of cooling

$$230 = (280 - 80)e^{-0.0048t} + 80$$

Substitute for T , T_0 , T_R , and r .

$$150 = 200e^{-0.0048t}$$

Subtract 80 from each side.

$$0.75 = e^{-0.0048t}$$

Divide each side by 200.

$$\ln 0.75 = \ln e^{-0.0048t}$$

Take natural log of each side.

$$-0.2877 \approx -0.0048t$$

$\ln e^x = \log_e e^x = x$

$$60 \approx t$$

Divide each side by -0.0048 .

► You have to wait about 60 minutes until you can continue driving.



GUIDED PRACTICE for Examples 2 and 3

Solve the equation.

4. $2^x = 5$

5. $7^{9x} = 15$

6. $4e^{-0.3x} - 7 = 13$