EXAMPLE 4 Use the change-of-base formula

Evaluate $\log_3 8$ using common logarithms and natural logarithms.

Solution

Using common logarithms: $\log_3 8 = \frac{\log 8}{\log 3} \approx \frac{0.9031}{0.4771} \approx 1.893$

Using natural logarithms: $\log_3 8 = \frac{\ln 8}{\ln 3} \approx \frac{2.0794}{1.0986} \approx 1.893$

EXAMPLE 5 Use properties of logarithms in real life

SOUND INTENSITY For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the sound's intensity doubles. By how many decibels does the loudness increase?

Solution

Let I be the original intensity, so that $2I$ is the doubled intensity.

Increase in loudness $= L(2I) - L(I)$	**Write an expression.**
$= 10 \log \frac{2I}{I_0} - 10 \log \frac{I}{I_0}$	**Substitute.**
$= 10\left(\log \frac{2I}{I_0} - \log \frac{I}{I_0}\right)$	**Distributive property**
$= 10\left(\log 2 + \log \frac{I}{I_0} - \log \frac{I}{I_0}\right)$	**Product property**
$= 10 \log 2$	**Simplify.**
≈ 3.01	**Use a calculator.**

▶ The loudness increases by about 3 decibels.

✓ GUIDED PRACTICE for Examples 4 and 5

Use the change-of-base formula to evaluate the logarithm.

7. $\log_5 8$ **8.** $\log_8 14$ **9.** $\log_{26} 9$ **10.** $\log_{12} 30$

11. **WHAT IF?** In Example 5, suppose the artist turns up the volume so that the sound's intensity triples. By how many decibels does the loudness increase?