7.4 Evaluate Logarithms and Graph Logarithmic Functions

TEKS 2A.4.C, 2A.11.A, 2A.11.B, 2A.11.C

Before You evaluated and graphed exponential functions.

Now You will evaluate logarithms and graph logarithmic functions.

Why? So you can model the wind speed of a tornado, as in Example 4.

Key Vocabulary
- **logarithm of *y* with base *b***
- **common logarithm**
- **natural logarithm**

You know that $2^2 = 4$ and $2^3 = 8$. However, for what value of x does $2^x = 6$? Mathematicians define this x-value using a *logarithm* and write $x = \log_2 6$. The definition of a logarithm can be generalized as follows.

KEY CONCEPT *For Your Notebook*

Definition of Logarithm with Base *b*

Let b and y be positive numbers with $b \neq 1$. The **logarithm of *y* with base *b*** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in *logarithmic form* and the second is in *exponential form*.

EXAMPLE 1 Rewrite logarithmic equations

	Logarithmic Form	Exponential Form
a.	$\log_2 8 = 3$	$2^3 = 8$
b.	$\log_4 1 = 0$	$4^0 = 1$
c.	$\log_{12} 12 = 1$	$12^1 = 12$
d.	$\log_{1/4} 4 = -1$	$\left(\frac{1}{4}\right)^{-1} = 4$

Parts (b) and (c) of Example 1 illustrate two special logarithm values that you should learn to recognize. Let b be a positive real number such that $b \neq 1$.

Logarithm of 1

$\log_b 1 = 0$ because $b^0 = 1$.

Logarithm of *b* with Base *b*

$\log_b b = 1$ because $b^1 = b$.

GUIDED PRACTICE for Example 1

Rewrite the equation in exponential form.

1. $\log_3 81 = 4$ **2.** $\log_7 7 = 1$ **3.** $\log_{14} 1 = 0$ **4.** $\log_{1/2} 32 = -5$