## 73 Use Functions Involving $e$ <br> 2A.11.C, 2A.11.F

Before You studied exponential growth and decay functions.
Now
Why? You will study functions involving the natural base $e$. So you can model visibility underwater, as in Ex. 59.


Key Vocabulary - natural base $e$

The history of mathematics is marked by the discovery of special numbers such as $\pi$ and $i$. Another special number is denoted by the letter $e$. The number is called the natural base $\boldsymbol{e}$ or the Euler number after its discoverer, Leonhard Euler (1707-1783). The expression $\left(1+\frac{1}{n}\right)^{n}$ approaches $e$ as $n$ increases.

| $n$ | $10^{1}$ | $10^{2}$ | $10^{3}$ | $10^{4}$ | $10^{5}$ | $10^{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(\mathbf{1}+\frac{\mathbf{1}}{\boldsymbol{n}}\right)^{\boldsymbol{n}}$ | 2.59374 | 2.70481 | 2.71692 | 2.71815 | 2.71827 | 2.71828 |

## KEY CONCEPT

For Your Notebook

## The Natural Base $e$

The natural base $e$ is irrational. It is defined as follows:
As $n$ approaches $+\infty,\left(1+\frac{1}{n}\right)^{n}$ approaches $e \approx 2.718281828$.

## EXAMPLE 1 Simplify natural base expressions

## Simplify the expression.

a. $e^{2} \cdot e^{5}=e^{2+5}$
b. $\frac{12 e^{4}}{3 e^{3}}=4 e^{4-3}$
c. $\left(5 e^{-3 x}\right)^{2}=5^{2}\left(e^{-3 x}\right)^{2}$
$=e^{7}$
$=4 e$

$$
=25 e^{-6 x}=\frac{25}{e^{6 x}}
$$

## REVIEW

EXPONENTS
For help with properties of exponents, see p. 330.

## EXAMPLE 2 Evaluate natural base expressions

Use a calculator to evaluate the expression.

| $\quad$ Expression | Keystrokes | Display |
| :--- | :--- | :--- |
| a. $e^{4}$ | 2nd $\left.\left[\mathrm{e}^{x}\right] 4 \square\right)$ ENTER | 54.59815003 |
| b. $e^{-0.09}$ | 2nd $\left.\left.\left[\mathrm{e}^{x}\right](-)\right] .09 \square\right)$ ENTER | 0.9139311853 |

