

EXPONENTIAL DECAY MODELS When a real-life quantity decreases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$y = a(1 - r)^t$$

where a is the initial amount and r is the percent decrease expressed as a decimal. Note that the quantity $1 - r$ is the decay factor.



EXAMPLE 4 **Solve a Real-World Multi-Step Problem**

SNOWMOBILES A new snowmobile costs \$4200. The value of the snowmobile decreases by 10% each year.

- Write an exponential decay model giving the snowmobile's value y (in dollars) after t years. Estimate the value after 3 years.
- Graph the model.
- Use the graph to estimate when the value of the snowmobile will be \$2500.



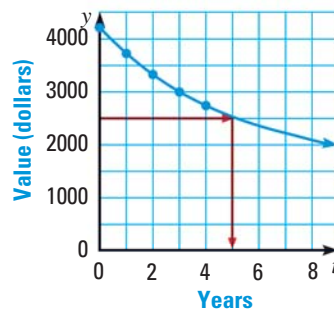
Solution

STEP 1 The initial amount is $a = 4200$ and the percent decrease is $r = 0.10$. So, the exponential decay model is:

$$\begin{aligned} y &= a(1 - r)^t && \text{Write exponential decay model.} \\ &= 4200(1 - 0.10)^t && \text{Substitute 4200 for } a \text{ and 0.10 for } r. \\ &= 4200(0.90)^t && \text{Simplify.} \end{aligned}$$

When $t = 3$, the snowmobile's value is $y = 4200(0.90)^3 = \$3061.80$.

STEP 2 The graph passes through the points $(0, 4200)$ and $(1, 3780)$. It has the t -axis as an asymptote. Plot a few other points. Then draw a smooth curve through the points.



STEP 3 Using the graph, you can estimate that the value of the snowmobile will be \$2500 after about 5 years.

AVOID ERRORS

Notice that the percent decrease, 10%, tells you how much value the snowmobile *loses* each year. The decay factor, 0.90, tells you what fraction of the snowmobile's value *remains* each year.



GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

4. $y = \left(\frac{1}{4}\right)^{x-1} + 1$

5. $y = 5\left(\frac{2}{3}\right)^{x+1} - 2$

6. $g(x) = -3\left(\frac{3}{4}\right)^{x-5} + 4$

7. **WHAT IF?** In Example 4, suppose the value of the snowmobile decreases by 20% each year. Write and graph an equation to model this situation. Use the graph to estimate when the value of the snowmobile will be \$2500.

8. **SNOWMOBILE** The value of a snowmobile has been decreasing by 7% each year since it was new. After 3 years, the value is \$3000. Find the original cost of the snowmobile.