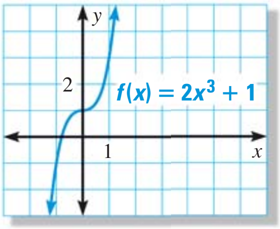
EXAMPLE 5 Find the inverse of a cubic function

Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

Solution

Graph the function f. Notice that no horizontal line intersects the graph more than once. So, the inverse of f is itself a function. To find an equation for f^{-1}, complete the following steps:



$f(x) = 2x^3 + 1$ **Write original function.**

$y = 2x^3 + 1$ **Replace $f(x)$ with y.**

$x = 2y^3 + 1$ **Switch x and y.**

$x - 1 = 2y^3$ **Subtract 1 from each side.**

$\frac{x-1}{2} = y^3$ **Divide each side by 2.**

$\sqrt[3]{\frac{x-1}{2}} = y$ **Take cube root of each side.**

▶ The inverse of f is $f^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$.

✓ GUIDED PRACTICE for Examples 4 and 5

Find the inverse of the function. Then graph the function and its inverse.

5. $f(x) = x^6, x \geq 0$

6. $g(x) = \frac{1}{27}x^3$

7. $f(x) = -\frac{64}{125}x^3$

8. $f(x) = -x^3 + 4$

9. $f(x) = 2x^5 + 3$

10. $g(x) = -7x^5 + 7$

EXAMPLE 6 Find the inverse of a power model

TICKET PRICES The average price P (in dollars) for a National Football League ticket can be modeled by

$$P = 35t^{0.192}$$

where t is the number of years since 1995. Find the inverse model that gives time as a function of the average ticket price.

Solution

$P = 35t^{0.192}$ **Write original model.**

$\frac{P}{35} = t^{0.192}$ **Divide each side by 35.**

$\left(\frac{P}{35}\right)^{1/0.192} = \left(t^{0.192}\right)^{1/0.192}$ **Raise each side to the power $\frac{1}{0.192}$.**

$\left(\frac{P}{35}\right)^{5.2} \approx t$ **Simplify. This is the inverse model.**