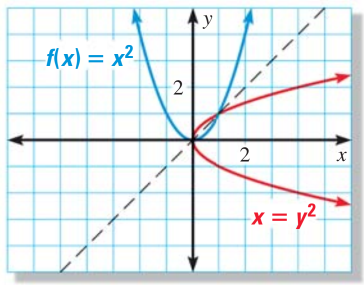
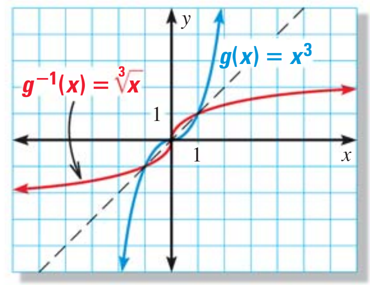
INVERSES OF NONLINEAR FUNCTIONS The graphs of the power functions $f(x) = x^2$ and $g(x) = x^3$ are shown below along with their reflections in the line $y = x$. Notice that the inverse of $g(x) = x^3$ is a function, but that the inverse of $f(x) = x^2$ is *not* a function.





If the domain of $f(x) = x^2$ is *restricted* to only nonnegative real numbers, then the inverse of f *is* a function.

EXAMPLE 4 Find the inverse of a power function

Find the inverse of $f(x) = x^2, x \geq 0$. Then graph f and f^{-1}.

Solution

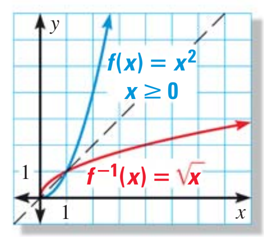
$f(x) = x^2$ — Write original function.

$y = x^2$ — Replace $f(x)$ with y.

$x = y^2$ — Switch x and y.

$\pm\sqrt{x} = y$ — Take square roots of each side.

The domain of f is restricted to nonnegative values of x. So, the range of f^{-1} must also be restricted to nonnegative values, and therefore the inverse is $f^{-1}(x) = \sqrt{x}$. (If the domain was restricted to $x \leq 0$, you would choose $f^{-1}(x) = -\sqrt{x}$.)



CHECK SOLUTION

You can check the solution of Example 4 by noting that the graph of $f^{-1}(x) = \sqrt{x}$ is the reflection of the graph of $f(x) = x^2, x \geq 0$, in the line $y = x$.

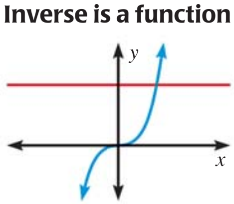
HORIZONTAL LINE TEST You can use the graph of a function f to determine whether the inverse of f is a function by applying the *horizontal line test*.

KEY CONCEPT *For Your Notebook*

Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function



Inverse is not a function