COMPOSITION OF FUNCTIONS Another operation that can be performed with two functions is *composition*.

READING
As with subtraction and division of functions, you need to be alert to the order of functions when they are composed. In general, $f(g(x))$ is not equal to $g(f(x))$.

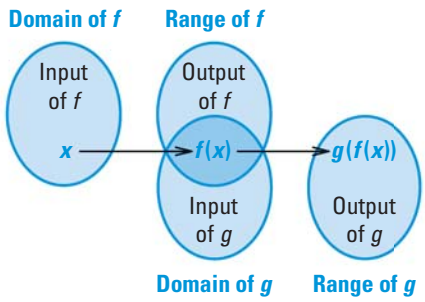
KEY CONCEPT *For Your Notebook*

Composition of Functions

The **composition** of a function g with a function f is:

$$h(x) = g(f(x))$$

The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.



TAKS

EXAMPLE 4 TAKS PRACTICE: Multiple Choice

Let $f(x) = 3x - 14$ and $g(x) = x^2 + 5$. What is the value of $g(f(4))$?

Ⓐ -9 Ⓑ -1 Ⓒ 1 Ⓓ 9

Solution

To evaluate $g(f(4))$, you first must find $f(4)$.

$$f(4) = 3(4) - 14 = -2$$

Then $g(f(4)) = g(-2) = (-2)^2 + 5 = 4 + 5 = 9$.

So, the value of $g(f(4))$ is 9.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 5 Find compositions of functions

Let $f(x) = 4x^{-1}$ and $g(x) = 5x - 2$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$

c. $f(f(x))$ **d.** the domain of each composition

Solution

a. $f(g(x)) = f(5x - 2) = 4(5x - 2)^{-1} = \dfrac{4}{5x - 2}$

b. $g(f(x)) = g(4x^{-1}) = 5(4x^{-1}) - 2 = 20x^{-1} - 2 = \dfrac{20}{x} - 2$

c. $f(f(x)) = f(4x^{-1}) = 4(4x^{-1})^{-1} = 4(4^{-1}x) = 4^0x = x$

d. The domain of $f(g(x))$ consists of all real numbers except $x = \dfrac{2}{5}$ because $g\left(\dfrac{2}{5}\right) = 0$ is not in the domain of f. (Note that $f(0) = \dfrac{4}{0}$, which is undefined.) The domains of $g(f(x))$ and $f(f(x))$ consist of all real numbers except $x = 0$, again because 0 is not in the domain of f.

AVOID ERRORS
You cannot always determine the domain of a composition from its equation. For instance, the domain of $f(f(x)) = x$ appears to be all real numbers, but it is actually all real numbers except zero.