EXAMPLE 6 **Simplify expressions involving variables**

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} = 4y^2$

b. $(27p^3q^{12})^{1/3} = 27^{1/3}(p^3)^{1/3}(q^{12})^{1/3} = 3p^{(3\,\cdot\,1/3)}q^{(12\,\cdot\,1/3)} = 3pq^4$

c. $\sqrt[4]{\dfrac{m^4}{n^8}} = \dfrac{\sqrt[4]{m^4}}{\sqrt[4]{n^8}} = \dfrac{\sqrt[4]{m^4}}{\sqrt[4]{(n^2)^4}} = \dfrac{m}{n^2}$

d. $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1\,-\,3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$

EXAMPLE 7 **Write variable expressions in simplest form**

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^{10}b^4c^5}$ **Factor out perfect fifth powers.**

$\qquad\qquad\quad = \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4}$ **Product property**

$\qquad\qquad\quad = ab^2c\sqrt[5]{4a^3b^4}$ **Simplify.**

AVOID ERRORS
You must multiply both the numerator *and* denominator of the fraction by y so that the value of the fraction does not change.

b. $\sqrt[3]{\dfrac{x}{y^8}} = \sqrt[3]{\dfrac{x \cdot y}{y^8 \cdot y}}$ **Make denominator a perfect cube.**

$\qquad\quad = \sqrt[3]{\dfrac{xy}{y^9}}$ **Simplify.**

$\qquad\quad = \dfrac{\sqrt[3]{xy}}{\sqrt[3]{y^9}}$ **Quotient property**

$\qquad\quad = \dfrac{\sqrt[3]{xy}}{y^3}$ **Simplify.**

EXAMPLE 8 **Add and subtract expressions involving variables**

Perform the indicated operation. Assume all variables are positive.

a. $\dfrac{1}{5}\sqrt{w} + \dfrac{3}{5}\sqrt{w} = \left(\dfrac{1}{5} + \dfrac{3}{5}\right)\sqrt{w} = \dfrac{4}{5}\sqrt{w}$

b. $3xy^{1/4} - 8xy^{1/4} = (3 - 8)xy^{1/4} = -5xy^{1/4}$

c. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2} = 12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2} = (12z - 3z)\sqrt[3]{2z^2} = 9z\sqrt[3]{2z^2}$

✓ **GUIDED PRACTICE**   for Examples 6, 7, and 8

Simplify the expression. Assume all variables are positive.

10. $\sqrt[3]{27q^9}$ **11.** $\sqrt[5]{\dfrac{x^{10}}{y^5}}$ **12.** $\dfrac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$ **13.** $\sqrt{9w^5} - w\sqrt{w^3}$