

RATIONAL EXPONENTS A rational exponent does not have to be of the form $\frac{1}{n}$. Other rational numbers such as $\frac{3}{2}$ and $-\frac{1}{2}$ can also be used as exponents. Two properties of rational exponents are shown below.

KEY CONCEPT

For Your Notebook

Rational Exponents

Let $a^{1/n}$ be an n th root of a , and let m be a positive integer.

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0$$

EXAMPLE 2 Evaluate expressions with rational exponents

Evaluate (a) $16^{3/2}$ and (b) $32^{-3/5}$.

Solution

Rational Exponent Form

a. $16^{3/2} = (16^{1/2})^3 = 4^3 = 64$

b. $32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(32^{1/5})^3} = \frac{1}{2^3} = \frac{1}{8}$

Radical Form

$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$

$32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3} = \frac{1}{8}$

AVOID ERRORS

Be sure to use parentheses to enclose a rational exponent: $9^{(1/5)} \approx 1.552$. Without them, the calculator evaluates a power and then divides: $9^{1/5} = 1.8$.

EXAMPLE 3 Approximate roots with a calculator

Expression	Keystrokes	Display
a. $9^{1/5}$	9 \wedge (1 \div 5) ENTER	1.551845574
b. $12^{3/8}$	12 \wedge (3 \div 8) ENTER	2.539176951
c. $(\sqrt[4]{7})^3 = 7^{3/4}$	7 \wedge (3 \div 4) ENTER	4.303517071

GUIDED PRACTICE for Examples 1, 2, and 3

Find the indicated real n th root(s) of a .

1. $n = 4, a = 625$

2. $n = 6, a = 64$

3. $n = 3, a = -64$

4. $n = 5, a = 243$

Evaluate the expression without using a calculator.

5. $4^{5/2}$

6. $9^{-1/2}$

7. $81^{3/4}$

8. $1^{7/8}$

Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.

9. $4^{2/5}$

10. $64^{-2/3}$

11. $(\sqrt[4]{16})^5$

12. $(\sqrt[3]{-30})^2$