6.1 Evaluate *n*th Roots and Use Rational Exponents

TEKS *a.1, 2A.2.A*

Before	You evaluated square roots and used properties of exponents.
Now	You will evaluate *n*th roots and study rational exponents.
Why?	So you can find the radius of a spherical object, as in Ex. 60.

Key Vocabulary
- ***n*th root of *a***
- **index of a radical**

You can extend the concept of a square root to other types of roots. For example, 2 is a cube root of 8 because $2^3 = 8$. In general, for an integer n greater than 1, if $b^n = a$, then b is an ***n*th root of *a***. An nth root of a is written as $\sqrt[n]{a}$ where n is the **index** of the radical.

You can also write an nth root of a as a power of a. If you assume the power of a power property applies to rational exponents, then the following is true:

$$(a^{1/2})^2 = a^{(1/2)\cdot 2} = a^1 = a$$
$$(a^{1/3})^3 = a^{(1/3)\cdot 3} = a^1 = a$$
$$(a^{1/4})^4 = a^{(1/4)\cdot 4} = a^1 = a$$

Because $a^{1/2}$ is a number whose square is a, you can write $\sqrt{a} = a^{1/2}$. Similarly, $\sqrt[3]{a} = a^{1/3}$ and $\sqrt[4]{a} = a^{1/4}$. In general, $\sqrt[n]{a} = a^{1/n}$ for any integer n greater than 1.

KEY CONCEPT *For Your Notebook*

Real *n*th Roots of *a*

Let n be an integer $(n > 1)$ and let a be a real number.

***n* is an even integer.**		***n* is an odd integer.**	
$a < 0$	No real nth roots.	$a < 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$	One real nth root: $\sqrt[n]{0} = 0$	$a = 0$	One real nth root: $\sqrt[n]{0} = 0$
$a > 0$	Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$a > 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$

EXAMPLE 1 Find *n*th roots

Find the indicated real *n*th root(s) of *a*.

a. $n = 3, a = -216$

b. $n = 4, a = 81$

Solution

a. Because $n = 3$ is odd and $a = -216 < 0$, -216 has one real cube root. Because $(-6)^3 = -216$, you can write $\sqrt[3]{-216} = -6$ or $(-216)^{1/3} = -6$.

b. Because $n = 4$ is even and $a = 81 > 0$, 81 has two real fourth roots. Because $3^4 = 81$ and $(-3)^4 = 81$, you can write $\pm\sqrt[4]{81} = \pm 3$ or $\pm 81^{1/4} = \pm 3$.