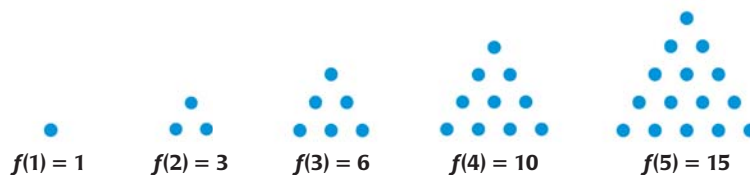


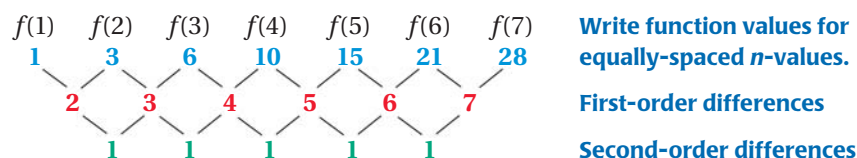
EXAMPLE 2 Find finite differences

The first five triangular numbers are shown below. A formula for the n th triangular number is $f(n) = \frac{1}{2}(n^2 + n)$. Show that this function has constant second-order differences.



Solution

Write the first several triangular numbers. Find the first-order differences by subtracting consecutive triangular numbers. Then find the second-order differences by subtracting consecutive first-order differences.



► Each second-order difference is 1, so the second-order differences are constant.



GUIDED PRACTICE for Examples 1 and 2

Write a cubic function whose graph passes through the given points.

- $(-4, 0), (0, 10), (2, 0), (5, 0)$
- $(-1, 0), (0, -12), (2, 0), (3, 0)$
- GEOMETRY** Show that $f(n) = \frac{1}{2}n(3n - 1)$, a formula for the n th pentagonal number, has constant second-order differences.

PROPERTIES OF FINITE DIFFERENCES In Example 2, notice that the function has degree two and that the second-order differences are constant. This illustrates the first of the following two properties of finite differences.

KEY CONCEPT

For Your Notebook

Properties of Finite Differences

- If a polynomial function $f(x)$ has degree n , then the n th-order differences of function values for equally-spaced x -values are nonzero and constant.
- Conversely, if the n th-order differences of equally-spaced data are nonzero and constant, then the data can be represented by a polynomial function of degree n .

The second property of finite differences allows you to write a polynomial function that models a set of equally-spaced data.