EXAMPLE 2 Find finite differences

The first five triangular numbers are shown below. A formula for the nth triangular number is $f(n) = \frac{1}{2}(n^2 + n)$. Show that this function has constant second-order differences.

$f(1) = 1$ $f(2) = 3$ $f(3) = 6$ $f(4) = 10$ $f(5) = 15$

Solution

Write the first several triangular numbers. Find the first-order differences by subtracting consecutive triangular numbers. Then find the second-order differences by subtracting consecutive first-order differences.

$f(1)$		$f(2)$		$f(3)$		$f(4)$		$f(5)$		$f(6)$		$f(7)$	
1		3		6		10		15		21		28	Write function values for equally-spaced n-values.
	2		3		4		5		6		7		First-order differences
		1		1		1		1		1			Second-order differences

▸ Each second-order difference is 1, so the second-order differences are constant.

✓ GUIDED PRACTICE for Examples 1 and 2

Write a cubic function whose graph passes through the given points.

1. $(-4, 0), (0, 10), (2, 0), (5, 0)$

2. $(-1, 0), (0, -12), (2, 0), (3, 0)$

3. GEOMETRY Show that $f(n) = \frac{1}{2}n(3n - 1)$, a formula for the nth pentagonal number, has constant second-order differences.

PROPERTIES OF FINITE DIFFERENCES In Example 2, notice that the function has degree two and that the second-order differences are constant. This illustrates the first of the following two properties of finite differences.

KEY CONCEPT *For Your Notebook*

Properties of Finite Differences

1. If a polynomial function $f(x)$ has degree n, then the nth-order differences of function values for equally-spaced x-values are nonzero and constant.
2. Conversely, if the nth-order differences of equally-spaced data are nonzero and constant, then the data can be represented by a polynomial function of degree n.

The second property of finite differences allows you to write a polynomial function that models a set of equally-spaced data.