5.9 Write Polynomial Functions and Models

TEKS a.3, 2A.1.B, 2A.3.B; P.3.B

Before You wrote linear and quadratic functions.

Now You will write higher-degree polynomial functions.

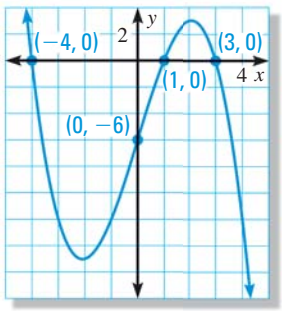
Why? So you can model launch speed, as in Example 4.

Key Vocabulary
- **finite differences**

You know that two points determine a line and that three points determine a parabola. In Example 1, you will see that four points determine the graph of a cubic function.

EXAMPLE 1 Write a cubic function

Write the cubic function whose graph is shown.



Solution

STEP 1 **Use** the three given x-intercepts to write the function in factored form.

$$f(x) = a(x + 4)(x - 1)(x - 3)$$

STEP 2 **Find** the value of a by substituting the coordinates of the fourth point.

$$-6 = a(0 + 4)(0 - 1)(0 - 3)$$

$$-6 = 12a$$

$$-\frac{1}{2} = a$$

▶ The function is $f(x) = -\frac{1}{2}(x + 4)(x - 1)(x - 3)$.

CHECK Check the end behavior of f. The degree of f is odd and $a < 0$. So $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$, which matches the graph. ✓

FINITE DIFFERENCES In Example 1, you found a function given its graph. Functions can also be written from a set of data using *finite differences*.

When the x-values in a data set are equally spaced, the differences of consecutive y-values are called **finite differences**. For example, some finite differences for the function $f(x) = x^2$ are shown below.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	← Values of $f(x)$ for equally-spaced x-values
1	4	9	16	
$4 - 1 = 3$	$9 - 4 = 5$	$16 - 9 = 7$		← Finite differences

The finite differences above are called *first-order differences*. You can also calculate higher-order differences, as shown in the next example.