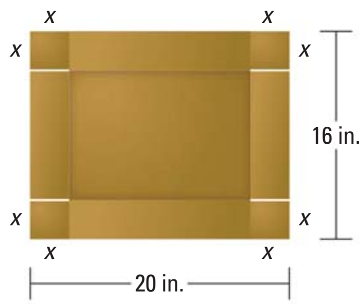
EXAMPLE 3 Maximize a polynomial model

ARTS AND CRAFTS You are making a rectangular box out of a 16-inch-by-20-inch piece of cardboard. The box will be formed by making the cuts shown in the diagram and folding up the sides. You want the box to have the greatest volume possible.

- How long should you make the cuts?
- What is the maximum volume?
- What will the dimensions of the finished box be?



Solution

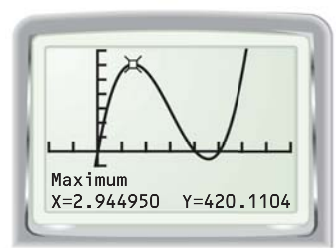
Write a verbal model for the volume. Then write a function.

Volume (cubic inches)	=	Length (inches)	·	Width (inches)	·	Height (inches)
V	=	$(20 - 2x)$	·	$(16 - 2x)$	·	x

$= (320 - 72x + 4x^2)x$ **Multiply binomials.**

$= 4x^3 - 72x^2 + 320x$ **Write in standard form.**

To find the maximum volume, graph the volume function on a graphing calculator, as shown at the right. Consider only the interval $0 < x < 8$ because this describes the physical restrictions on the size of the flaps.



From the graph, you can see that the maximum volume is about 420 and occurs when $x \approx 2.94$.

▶ You should make the cuts about 3 inches long. The maximum volume is about 420 cubic inches. The dimensions of the box with this volume will be about $x = 3$ inches by $x = 10$ inches by $x = 14$ inches.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the function. Identify the *x*-intercepts and the points where the local maximums and local minimums occur.

1. $f(x) = 0.25(x + 2)(x - 1)(x - 3)$
2. $g(x) = 2(x - 1)^2(x - 4)$
3. $h(x) = 0.5x^3 + x^2 - x + 2$
4. $f(x) = x^4 + 3x^3 - x^2 - 4x - 5$
5. **WHAT IF?** In Example 3, how do the answers change if the piece of cardboard is 10 inches by 15 inches?