5.8 Analyze Graphs of Polynomial Functions

TEKS 2A.4.B; P.1.D, P.1.E, P.3.B

Before You graphed polynomial functions by making tables.

Now You will use intercepts to graph polynomial functions.

Why? So you can maximize the volume of structures, as in Ex. 42.

Key Vocabulary
- local maximum
- local minimum

In this chapter you have learned that zeros, factors, solutions, and x-intercepts are closely related concepts. The relationships are summarized below.

CONCEPT SUMMARY — *For Your Notebook*

Zeros, Factors, Solutions, and Intercepts

Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ be a polynomial function. The following statements are equivalent.

Zero: k is a zero of the polynomial function f.

Factor: $x - k$ is a factor of the polynomial $f(x)$.

Solution: k is a solution of the polynomial equation $f(x) = 0$.

x-intercept: If k is a real number, k is an x-intercept of the graph of the polynomial function f. The graph of f passes through $(k, 0)$.

EXAMPLE 1 Use x-intercepts to graph a polynomial function

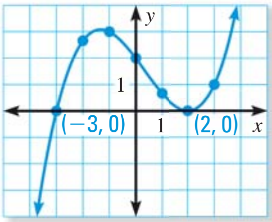
Graph the function $f(x) = \frac{1}{6}(x + 3)(x - 2)^2$.

Solution

STEP 1 **Plot** the intercepts. Because -3 and 2 are zeros of f, plot $(-3, 0)$ and $(2, 0)$.

STEP 2 **Plot** points between and beyond the x-intercepts.

x	-2	-1	0	1	3
y	$\frac{8}{3}$	3	2	$\frac{2}{3}$	1



STEP 3 **Determine** end behavior. Because f has three factors of the form $x - k$ and a constant factor of $\frac{1}{6}$, it is a cubic function with a positive leading coefficient. So, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.