EXAMPLE 4 Use Descartes' rule of signs

Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for $f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$.

Solution

$$f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$$

The coefficients in $f(x)$ have **3 sign changes**, so f has 3 or 1 positive real zero(s).

$$f(-x) = (-x)^6 - 2(-x)^5 + 3(-x)^4 - 10(-x)^3 - 6(-x)^2 - 8(-x) - 8$$

$$= x^6 + 2x^5 + 3x^4 + 10x^3 - 6x^2 + 8x - 8$$

The coefficients in $f(-x)$ have **3 sign changes**, so f has 3 or 1 negative real zero(s).

The possible numbers of zeros for f are summarized in the table below.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	3	0	6
3	1	2	6
1	3	2	6
1	1	4	6

✓ GUIDED PRACTICE for Example 4

Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for the function.

9. $f(x) = x^3 + 2x - 11$

10. $g(x) = 2x^4 - 8x^3 + 6x^2 - 3x + 1$

APPROXIMATING ZEROS All of the zeros of the function in Example 4 are irrational or imaginary. Irrational zeros can be approximated using technology.

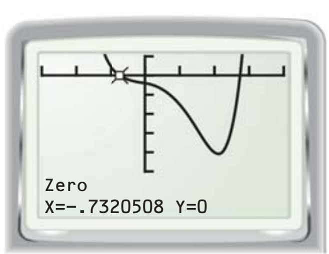
EXAMPLE 5 Approximate real zeros

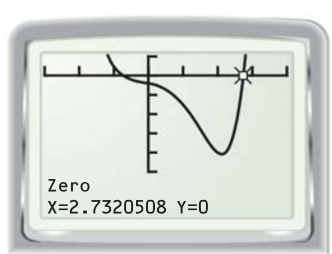
Approximate the real zeros of $f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$.

Solution

Use the *zero* (or *root*) feature of a graphing calculator, as shown below.

ANOTHER WAY
In Example 5, you can also approximate the zeros of f using the calculator's *trace* feature. However, this generally gives less precise results than the *zero* (or *root*) feature.





▸ From these screens, you can see that the zeros are $x \approx -0.73$ and $x \approx 2.73$.