EXAMPLE 2 Find the zeros of a polynomial function

Find all zeros of $f(x) = x^5 - 4x^4 + 4x^3 + 10x^2 - 13x - 14$.

Solution

STEP 1 **Find** the rational zeros of f. Because f is a polynomial function of degree 5, it has 5 zeros. The possible rational zeros are ± 1, ± 2, ± 7, and ± 14. Using synthetic division, you can determine that -1 is a zero repeated twice and 2 is also a zero.

STEP 2 **Write** $f(x)$ in factored form. Dividing $f(x)$ by its known factors $x + 1$, $x + 1$, and $x - 2$ gives a quotient of $x^2 - 4x + 7$. Therefore:

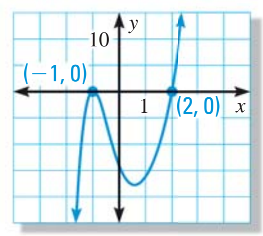
$$f(x) = (x + 1)^2(x - 2)(x^2 - 4x + 7)$$

STEP 3 **Find** the complex zeros of f. Use the quadratic formula to factor the trinomial into linear factors.

$$f(x) = (x + 1)^2(x - 2)\left[x - (2 + i\sqrt{3})\right]\left[x - (2 - i\sqrt{3})\right]$$

▶ The zeros of f are -1, -1, 2, $2 + i\sqrt{3}$, and $2 - i\sqrt{3}$.

BEHAVIOR NEAR ZEROS The graph of f in Example 2 is shown at the right. Note that only the *real* zeros appear as x-intercepts. Also note that the graph is tangent to the x-axis at the repeated zero $x = -1$, but crosses the x-axis at the zero $x = 2$. This concept can be generalized as follows:



- When a factor $x - k$ of a function f is raised to an odd power, the graph of f crosses the x-axis at $x = k$.
- When a factor $x - k$ of a function f is raised to an even power, the graph of f is tangent to the x-axis at $x = k$.

✓ GUIDED PRACTICE for Example 2

Find all zeros of the polynomial function.

3. $f(x) = x^3 + 7x^2 + 15x + 9$

4. $f(x) = x^5 - 2x^4 + 8x^2 - 13x + 6$

REVIEW COMPLEX NUMBERS
For help with complex conjugates, see p. 278.

COMPLEX CONJUGATES Also in Example 2, notice that the zeros $2 + i\sqrt{3}$ and $2 - i\sqrt{3}$ are complex conjugates. This illustrates the first theorem given below. A similar result applies to irrational zeros of polynomial functions, as shown in the second theorem below.

KEY CONCEPT *For Your Notebook*

Complex Conjugates Theorem

If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.

Irrational Conjugates Theorem

Suppose f is a polynomial function with rational coefficients, and a and b are rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.