

5.7 Apply the Fundamental Theorem of Algebra

TEKS

2A.2.A, 2A.8.B;
P.1.D, P.3.B

Before

You found zeros using the rational zero theorem.

Now

You will classify the zeros of polynomial functions.

Why?

So you can determine boat speed, as in Example 6.



Key Vocabulary

- **repeated solution**
- **irrational conjugates**, p. 267
- **complex conjugates**, p. 278

The equation $x^3 - 5x^2 - 8x + 48 = 0$, which becomes $(x + 3)(x - 4)^2 = 0$ when factored, has only two distinct solutions: -3 and 4 . Because the factor $x - 4$ appears twice, however, you can count the solution 4 twice. So, with 4 counted as a **repeated solution**, this *third-degree* equation has *three* solutions: -3 , 4 , and 4 .

The previous result is generalized by the *fundamental theorem of algebra*, first proved by the German mathematician Karl Friedrich Gauss (1777–1855).

KEY CONCEPT

For Your Notebook

The Fundamental Theorem of Algebra

Theorem: If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary: If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as 2 solutions, each solution repeated three times is counted as 3 solutions, and so on.

The corollary to the fundamental theorem of algebra also implies that an n th-degree polynomial function f has exactly n zeros.

EXAMPLE 1 Find the number of solutions or zeros

- How many solutions does the equation $x^3 + 5x^2 + 4x + 20 = 0$ have?
- How many zeros does the function $f(x) = x^4 - 8x^3 + 18x^2 - 27$ have?

Solution

- Because $x^3 + 5x^2 + 4x + 20 = 0$ is a polynomial equation of degree 3, it has three solutions. (The solutions are -5 , $-2i$, and $2i$.)
- Because $f(x) = x^4 - 8x^3 + 18x^2 - 27$ is a polynomial function of degree 4, it has four zeros. (The zeros are -1 , 3 , 3 , and 3 .)



GUIDED PRACTICE for Example 1

- How many solutions does the equation $x^4 + 5x^2 - 36 = 0$ have?
- How many zeros does the function $f(x) = x^3 + 7x^2 + 8x - 16$ have?