EXAMPLE 3 Find zeros when the leading coefficient is not 1

Find all real zeros of $f(x) = 10x^4 - 11x^3 - 42x^2 + 7x + 12$.

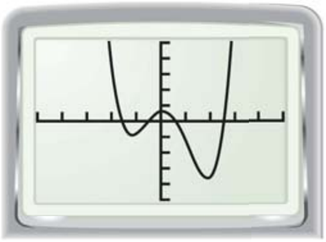
Solution

STEP 1 **List** the possible rational zeros of f: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$, $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{12}{5}, \pm\frac{1}{10}, \pm\frac{3}{10}$

STEP 2 **Choose** reasonable values from the list above to check using the graph of the function. For f, the values

$$x = -\frac{3}{2},\ x = -\frac{1}{2},\ x = \frac{3}{5},\ \text{and } x = \frac{12}{5}$$

are reasonable based on the graph shown at the right.



STEP 3 **Check** the values using synthetic division until a zero is found.

$$\begin{array}{r|rrrrr} -\frac{3}{2} & 10 & -11 & -42 & 7 & 12 \\ & & -15 & 39 & \frac{9}{2} & -\frac{69}{4} \\ \hline & 10 & -26 & -3 & \frac{23}{2} & -\frac{21}{4} \end{array}$$

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 10 & -11 & -42 & 7 & 12 \\ & & -5 & 8 & 17 & -12 \\ \hline & 10 & -16 & -34 & 24 & 0 \end{array}$$

$-\frac{1}{2}$ **is a zero.**

STEP 4 **Factor** out a binomial using the result of the synthetic division.

$f(x) = \left(x + \frac{1}{2}\right)(10x^3 - 16x^2 - 34x + 24)$ **Write as a product of factors.**

$= \left(x + \frac{1}{2}\right)(2)(5x^3 - 8x^2 - 17x + 12)$ **Factor 2 out of the second factor.**

$= (2x + 1)(5x^3 - 8x^2 - 17x + 12)$ **Multiply the first factor by 2.**

STEP 5 **Repeat** the steps above for $g(x) = 5x^3 - 8x^2 - 17x + 12$. Any zero of g will also be a zero of f. The possible rational zeros of g are:

$$x = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{12}{5}$$

The graph of g shows that $\frac{3}{5}$ may be a zero. Synthetic division shows that $\frac{3}{5}$ *is* a zero and $g(x) = \left(x - \frac{3}{5}\right)(5x^2 - 5x - 20) = (5x - 3)(x^2 - x - 4)$. It follows that:

$$f(x) = (2x + 1) \cdot g(x) = (2x + 1)(5x - 3)(x^2 - x - 4)$$

STEP 6 **Find** the remaining zeros of f by solving $x^2 - x - 4 = 0$.

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$ **Substitute 1 for *a*, −1 for *b*, and −4 for *c* in the quadratic formula.**

$x = \frac{1 \pm \sqrt{17}}{2}$ **Simplify.**

▶ The real zeros of f are $-\frac{1}{2}, \frac{3}{5}, \frac{1 + \sqrt{17}}{2}$, and $\frac{1 - \sqrt{17}}{2}$.